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# Comparing Traditional Statistical Models with Neural Network Models: The Case of the Relation of Human Performance Factors to the Outcomes of Military Combat 

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COMPARING TRADITIONAL STATISTICAL MODELS WITH NEURAL NETWORK MODELS: THE CASE OF THE RELATION OF HUMAN PERFORMANCE FACTORS TO THE OUTCOMES OF MILITARY COMBAT

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DOCTOR OF PHILOSOPHY<br>ENGINEERING MANAGEMENT<br>OLD DOMINION UNIVERSITY<br>December, 1995

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ABSTRACT<br>COMPARING TRADITIONAL STATISTICAL MODELS WITH NEURAL NETWORK MODELS: THE CASE OF THE RELATION OF HUMAN PERFORMANCE FACTORS TO THE OUTCOMES OF MILITARY COMBAT<br>William Oliver Hedgepeth<br>Old Dominion University, 1995<br>Director: Dr. Derya A. Jacobs


#### Abstract

Statistics and neural networks are analytical methods used to learn about observed experience. Both the statistician and neural network researcher develop and analyze data sets, draw relevant conclusions, and validate the conclusions. They also share in the challenge of creating accurate predictions of future events with noisy data.

Both analytical methods are investigated. This is accomplished by examining the veridicality of both with real system data. The real system used in this project is a database of 400 years of historical military combat. The relationships among the variables represented in this database are recognized as being hypercomplex and nonlinear.

The historical database was investigated from two paradigms. Paradigm I states that predicting the winner of combat can be based on post-combat personnel losses. Paradigm II states that predicting the winner can be based


on pre-combat initial conditions of personnel strength and skill factors.

The results give evidence that traditional statistical methods may provide greater accuracy in predictions when the data is clean or filtered (perfect) than when it is noisy and unfiltered (imperfect). Neural networks, on the other hand, may provide greater accuracy for the same predictions when the data is left imperfect than when it is cleaned up and filtered (perfect).

## DEDICATION

## To Elizabeth.

Obtaining a terminal degree is a path one takes toward learning about learning. But, the journey begins with a helping hand from those who have taken that path. The one who helped guide me onto that path and set the direction was Dr. Derya A. Jacobs. I will always be thankful for her patience and guidance.

Dr. Laurence D. Richards has a special thanks for teaching me how to open my eyes to a lifelong view of learning. Dr. Billie M. Reed was always there when I needed a sympathetic ear, from my first course of instruction in Engineering Management, to the last course I took prior to initiating my Ph.D. research. Alan Pope's initial energy and neural network advice helped formulate the research scope. Dr. Mark Scerbo provided his time and his talent to help keep my research focused.

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## CHAPTER 1

INTRODUCTION

The prediction of outcomes of military combat is a classic problem. Examples of these outcomes may be which side wins a battle, how many casualties occur, or which route is taken by a convoy of ships. Within the field of operations research, these and similar problems have been tackled by traditional statistical approaches for over 50 years. However, a problem arises in creating these predictions when the number of factors involved in the outcome is large, or when the relationships among the factors are complex and uncertain (Davis 1995). In addition, when examining historical databases of combat situations, there can be missing values within the data sets or variables. Traditional statistical methods often exclude an entire treatment case, e.g., a battle, when a missing data value is detected. These factors make the problem an excellent candidate for using an alternative approach to traditional statistical methods. One possible alternative is artificial neural networks. During this study, an artificial neural network was developed for comparison with a traditional statistical method. Both methods were used to examine their veridicality in the prediction of the outcomes
of combat situations, battles or wars, based on an authoritative historical combat database.

## Problem Statement

Military combat human-machine interactions may exhibit system hypercomplexity between initial combat conditions and predicted outputs (Geeraerts 1994). A consequence of this hypercomplexity is a high degree of uncertainty in the data that describes these interactions. In the past, military analysts dealt with this uncertainty by giving military decision makers predictions based on more "perfect," i.e., filtered and hard data, such as the number of personnel and equipment engaged in combat and the attrition rates of personnel and equipment. However, with the increased power of today's computer systems, it is now feasible to help decision makers explore an expanded set of possible battle or war outcomes by using this uncertainty, and by using more "imperfect" data, i.e., data with missing values and soft data, such as morale of personnel and leadership skills (Davis 1995; Arquilla 1992).

Artificial neural networks have begun to demonstrate some robust abilities in the analysis of complex data that has eluded traditional statistical approaches in providing accurate predictions of future events (Davis 1995; Cheng and Titterington 1994a; Cleckner 1994).

Therefore, the problem for this research is that the predictive capabilities of traditional statistical models may not be as robust when applied to noisy and incomplete data as they are when applied to clean, or filtered data, and combat data that focuses on skill-based and human factors tend to be noisy and incomplete. The human factors used in this database are qualitative measures of human performance in combat, such as leadership, morale, training, initiative, and combat effectiveness.

## Research Obiective

The need for this research comes from the convergence of several trends in rethinking how to analyze complex problems (Cheng and Titterington 1994a; Sharda 1994; Arquilla 1992; Morrison 1992; Padgett and Roppel 1992; White 1989). These are summarized as:
(1) uncertain or soft data may contain causal patterns of behavior different from those contained in clean or hard data;
(2) traditional statistical methods may have difficulty anaiyzing uncertain, missing, or soft data;
(3) computerized neural network algorithms are beginning to solve complex, nonlinear problems accurately; and,
(4) the credible use of neural networks needs further development.

The specific objective of this research was to investigate and compare the use of an artificial neural network and a traditional statistical approach in the analysis of large, complex databases for prediction purposes. A historical combat database of 660 battles and wars, spanning approximately 400 years, was used to design, train and test the statistical and neural network models. These two models were used to predict the winners of the 660 combat engagements. The winner, as either the attacker or defender, is an a priori variable defined by the historians who created the database. Thus, the two analytic models were trained to recognize the relation between multiple input variables describing each combat engagement, and the one output variable, which is the winner of each engagement.

This research should then be considered as a comparison of the inference capabilities of an artificial neural network and a traditional statistical model using a mix of qualitative and quantitative data. As such, this study should contribute to addressing the research need and objective by meeting the following goals:
(1) to identify new causal patterns in uncertain or soft data;
(2) to help fill the statistical gap when analyzing uncertain, missing, or soft data;
(3) to demonstrate a neural network's capabilities with a complex and nonlinear problem; and,
(4) to examine a potentially credible use of neural networks.

## Research_lypotheses

From the objective and need, the following two hypotheses were formulated for this research:
(1) There is a significant difference in the accuracy of model predictions of combat winners when based on input data that is clean and filtered (perfect) versus input data that is noisy and unfiltered (imperfect).
(1a) The data type that gives higher prediction accuracy for traditional statistical models (i.e., perfect data) is different from the data type that gives higher prediction accuracy for neural network models (i.e., imperfect data).
(2) The accuracy of the predictions of combat winners based on attrition data (i.e., combat casualties, which is Paradigm I) is significantly different from the accuracy of predictions based on strength and skill data (i.e., human factors, environment, force description, and doctrine and operations, which is Paradigm II).

## CHAPTER 2

## REVIEH OF LITERATURE


#### Abstract

Finding solutions to prediction problems involving nonlinear human factors and using combat performance data may be accomplished with statistical and neural network methods. There are decades of published research that describe statistical methods for analyzing a variety of such historic combat data (Davis 1995; Arquilla 1992; Helmbold 1987; Dupuy 1979; Stockfisch 1975; Bonder 1971). However, the literature on the use of neural networks as an alternative or complementary approach to these traditional statistical approaches is only beginning to be reported (Jacobs and Hedgepeth 1995; Kilmer 1995; Cheng and Titterington 1994a, 1994b; Cleckner 1994; Kilmer 1994a, 1994b, 1993; Sharda 1994; Bui, Dryer and Laskowski 1992; Eldridge 1992; Morrison 1992; Padgett and Roppel 1992). These reports indicate that neural networks may outperform traditional statistical methods when the data is composed of a large number of quantitative and qualitative variables (Cheng and Titterington 1994a). Thus, this study is a case of how traditional statistical and neural network methods can be used to analyze quantitative and qualitative data,


and specifically, to find relations between human performance factors and the outcomes of military combat.

## Statistical Methods Used In Combat Analysis

Arquilla (1992) analyzed historic combat data using logistic regression to identify causal patterns that might explain why some battles are won and some are lost. He found sufficient information from this statistical analysis to support the presence of causal patterns that cannot be attributed to chance.

Arquilla used statistical methods to try to find clues to the faulty human performance and behavior that could be causal for battle wins or losses. He analyzed the skillbased or soft data, such as data on technology and perceptions of power differentials between the combatants. His hypothesis rested on the belief that skill, rather than strength, was the dominant factor in combat, and that the relationship of the battle wins and losses to skill and strength is nonlinear.

Helmbold (1987) analyzed a similar historic database and discovered a relationship between casualty ratios and the probability of which side of a battle would win. The Helmbold approach used logistic regression methods. As such, he believes he has found a fundamental relationship in this historic combat data. However, Helmbold reports that the use of logistic regression is not very robust and is
subject to influence by errors in the imperfect database (Helmbold 1987), where the imperfect database contains missing data values.

McQuie (1988) examined a similar historic combat database by categorizing the data from the battle histories. He established standard characteristics, e.g., statistical means, for different data elements and compared them to data generated by computer wargames for any differences. He validated the standard characteristics by determining if they fell within a subjectively acceptable range of values (McQuie 1988).

Allen (1992) explored the use of a similar historic combat database for predicting the results of battles. Allen's methodology was to vary the strength of weapon characteristics as well as soft data elements such as environmental factors, terrain and weather, to calculate combat losses for each weapon. The methodology focused on the synergistic aspects of selected weapons used. He presented an advance in the state of the art in modeling combat situations by accounting for the combined weapons effects, which are frequently absent from combat models that employ single weapon effects (Allen 1992).

Trevor Dupuy is credited with creating the first view of combat causal effects that broke with traditional statistical and combat modeling and simulation viewpoints (Davis 1995). He created a complex analytic model, the Quantified Judgment Model or QJM, that predicted combat
outcomes as well as casualties. This break with traditional statistical reasoning used not only the force strength combat data, but also many soft data items, such as morale and surprise of the different forces engaged in combat. However, Dupuy's work was widely ignored and rejected by the military community from the 1970s to the early 1990s (Davis 1995; Davis and Blumenthal 1991). It is Dupuy's database, after being reviewed and authenticated over a 10 -year period of time by the United States Army, that is the database for this research. And, whereas Dupuy relied on the analytic statistical models available to create his QJM, this research goes to the next step of using robust neural networks that can account for higher-order interactions.

Helmbold (1987) cautions about examining incredibly complex political and military problems with statistics, which is part of the criticism directed toward Dupuy's work. Similarly, DeWeerd (1979) cautions that evaluating a battle quantitatively is unlikely to produce any causal relations.

Simon (1990) prescribes going beyond the number crunching ability of computerized models, such as those used by Helmbold and Allen. His suggestion is to substitute symbolic data for numeric data, which gets closer to the concept of Dupuy. Kilmer (1995) echoes Simon's (1990) exclamation about the need to go beyond brute force statistical analysis to more intelligent approximations of battles, if progress beyond the limitations of statistical combat modeling is to be reached.

Busse (1971) compared the statistical analysis of Lanchester combat equations to actual combat to show that the equations can fit actual combat, proving a link between battle winners and personnel casualties. This parallels the work of Helmbold. However, Busse cautions that statistical analyses are dependent on the veracity of the historic battle data. But, Dupuy (1983) argued that all data were potentially valuable and integrated the entire database into his analytic model, irrespective of judgments of its veracity.

Goldhamer (1979a) cautions that using history to make simple generalizations that accurately predict battle outcomes is unwise, which takes issue with the work of Arquilla, Helmbold and Dupuy. But, this caution has served as a challenge to the operations research community to continue to experiment with simple generalizations.

Stockfisch (1975) summarizes the state of combat models by indicating that they are of questionable worth, due to the lack of empirical study of historic battles. But, he sees the statistical analysis of historic combat by researchers such as Arquilla and Dupuy as helping bridge the gap between theory and fact with respect to how human performance affects combat. Stockfisch recognizes that there may be different causal relations derived from combat data when many different variables, that are both quantitative and qualitative, are analyzed.

## Neural Networks and Combat Analysis

Neural networks are nonlinear computational nodes operating in parallel and formed into a pattern that mimic biological neural networks. These nodes are connected through a weighting algorithm that determines what signal is passed from one node to another. The weights and signals are adaptive due to the recursive process of training a neural network. The result is a network algorithm or model that can react to, or observe, a stimulus and produce an outcome (Lippmann 1991; Nelson and Illingworth 1990).

Within historic combat databases, there is noise that is typical of human factors or human performance databases. This noise tends to limit the analytical value of traditional statistical analyses. But, it is this limitation, the noise within the data, and the difficulty of performing trend analysis, that is a motivating criterion for the use of neural networks. Some analysts indicate there is limited or no value in looking at such raw data with statistical models (McQuie 1988). This may not be valid using neural networks which have demonstrated the ability to examine raw, unfiltered data and find significant, and often new, patterns or causal links in some prediction models (Nelson and Illingworth 1990).

Current research indicates that backpropagation neural networks outperform other neural network algorithms for analyzing complex, uncertain data. For example, there are
documented advantages in using neural networks to model combat battles and potentially to replace combat simulations (Kilmer 1995; Kilmer, Smith and Shuman 1994; Launsby and Hallowell 1994; Sharda 1994; Kilmer and Smith 1993; Caudill and Butler 1992; Eldridge 1992).

One negative side of neural networks is the lack of any guarantee of producing a significant causal result. Barto (1993) issues a strong warning that neural network capabilities are currently in the exploratory stage for use in solving nonlinear problems. But, he suggests that one reason neural networks are becoming acceptable to engineers is that they are being applied to a wider class of problems (Barto 1993).

## The Bridge Between Statistics and Neural Networks

In 1994, statisticians and neural network researchers began to map interfaces between statistical and neural network perspectives and fundamental principles (Cheng and Titterington 1994a; Kilmer 1994a, 1994b). Neural networks were shown to have a mathematical structure similar to regression (Cheng and Titterington 1994a). For example, Kilmer (1995, 1994a, 1994b) indicates success in using neural networks to approximate the capabilities of combat simulations. He reports that neural networks required fewer assumptions and used noisy, or less precise, data. He also indicated that neural networks have structural similarities
with nonlinear least squares regression. Similarly, Eldridge (1992) demonstrated the success of a neural network to learn actual tank routes from battlefield test data and to produce accurate predictions of other tank route decisions made by commanders. And, Sharda (1994) demonstrated that a neural network could outperform human decision makers in predicting the outcome of simulated battles.

A difference between statistical regression methods and neural network methods is the freedom that neural networks offer to look at more data in different ways. A caution is that neural networks may need more input values for training (building) them than statistical regression does (Kilmer 1995; Sharda 1994; Kilmer and Smith 1993; Eldridge 1992). Another difference is that error statistics are not as easily derived from neural networks as they are from traditional statistical regressions (Kilmer, Smith and Shuman 1994).

While the data requirements for neural networks and statistical methods are different, a common problem area for both is the difficulty they encounter when there is a lack of input data (Kilmer and Smith 1993).

Many of the limitations identified and cautions expressed with respect to historic combat database analyses were based on a viewpoint from over a decade ago, when computers were not as powerful as they are today. Thus, the gap between traditional statistical and neural network
methods needs further exploration. The bridge between these methods, at this time, does seem to be based on the power of the computer. As such, new statistical methods may develop as new neural network methods are developing.

CHAPTER 3
METHODOLOGY

This research evaluates and compares the performance of artificial neural networks and statistical approaches for the prediction of combat outcomes based on historical battles. This evaluation requires the specification of the models, a data stratification strategy for building and testing these models, and a logic for assessing the performance of the models.

The experimentation process is divided into six tasks and discussed in the following sections.

Task 1: Planning and Data Collection

The historic combat database used in this research is provided by the United States Army Concepts Analysis Agency. This historic database presents treatment cases for battles and wars. Battles and wars can further be categorized as land or land-sea. The number of battles and wars that comprise the database are 660 , spanning a time period of approximately 400 years. Each battle and war has 41 possible variables that describe initial and final battle conditions.

Variables used in this experiment are categorized by two paradigms, both linked to prediction of the winner of the battles or wars. In Paradigm I, following Helmbold's (1987) research, the winners of combat battles and wars are predicted using a ratio of casualties:

$$
\begin{equation*}
C R=A C / D C \tag{1}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{CR}=\text { casualty ratio, } \\
& \mathrm{AC}=\text { number of attacker casualties, and } \\
& \mathrm{DC}=\text { number of defender casualties. }
\end{aligned}
$$

In Paradigm II, following Arquilla's (1992) research, the winners of combat battles and wars are predicted using data which includes soft data, such as human factors, skill, technology, terrain, and tactics, as well as hard data, such as force strength.

Helmbold (1987) and Arquilla (1992) both used logistic regression analysis in their research to determine the combat winners. Therefore, a rationale for the use of logistic regression (LR), or logit, as the traditional statistical method of analysis, is to keep constant what appears as a common analytic tool. LR is also appropriate for this type of analysis since the results can be coded as either a 1 or 0 . This 1 and 0 code is needed in the
research, since a win for the attacker force is represented with a value of 1 , and a loss by the attacker force is represented with a value of 0 .

There are over 100 artificial neural network algorithms for different types of problems. In the literature, backpropagation neural networks are shown to be successful in learning pattern recognition and time series analysis for prediction activities and problems. Therefore, a backpropagation neural network (BNN) algorithm was selected as the artificial neural network in this research.

The experimental plan relies on the use of $L R$ and BNN mathematical models for the two paradigms as shown in Table 1. The variable types for the two paradigms have been discussed previously.

The data used to examine each paradigm depicted in Table 1 is categorized into two types relating to the quality of the data. Quality is defined as a degree of battle or war data accuracy and completeness as reported by historians and military analysts. For purposes of this study, the quality of the data sets is defined as either perfect or imperfect.

Imperfect data refers to missing or uncertain data values for a particular battle or war. For example, of the 41 variables that describe a battle, an imperfect battle data set would be one with missing values, such as no value for the defender's number of artillery tubes. Conversely, a perfect battle data set would be one where all 41 variables


Table 1. Experimental Plan Showing Paradigms I and II and
Model Types.
have values, that is, no missing or unknown values. After analysis of the data, it was decided to create three subsets of data as described below:

- Data Set A: Includes 149 conflicts from 1600 to 1812.
- Data Set B: Includes 511 conflicts from 1812 to 1982.
- Data Set C: Includes 660 conflicts from 1600 to 1982 (Data Set A + Data Set B).

The breakpoint of 1812 was chosen based on the increased influence of technology on the battlefield from that time forward (Arquilla 1992; Helmbold 1987). Thus, the 149 conflicts of Data Set A can be considered as conflicts involving low technology. For example, from 1600-1812, horse cavalry was an important factor on the battlefield. The 511 conflicts of Data Set $B$ can be considered as conflicts involving high technology. For example, from 1812-1982, different individual weapons and long range weapons were introduced to the battlefield. Data Set C combines both Data Set A and Data Set B.

The analysis models, $L R$ and $B N N$, developed for these three data sets, to be used with Paradigm I and II, are shown in Table 2. The values entered into this table are the percentages of correctly predicted battle winners.

| Data Set | IR | BNN | IR | BNN |
| :---: | :---: | :---: | :---: | :---: |
| A | LRA (I) | BNNA (I) | LRA (II) | BNNA (II) |
| B | IRB (I) | BNNB (I) | IRB(II) | BNNB (II) |
| C | LRC (I) | BNNC (I) | LRC (II) | BNNC (II) |

Table 2. Experiment Design.

## Task 2: Determining Statistical Aloorithm

The algorithm for LR uses a threshold value (0.5) that when reached produces a value or 1 or 0 as the predicted outcome. This threshold is depicted as the midpoint of the $S$ curve created by $L R$, which produces a range of values from 0 to 1. Additionally, the LR produces other output parameters - number of data items examined, maximum likelihood estimates, and standard errors, which can be used to determine the statistical significance of the LR results.

The general form of the logistic function used is the univariate binary case defined by the equation:

$$
\begin{equation*}
P(A W)=\operatorname{EXP}(a+b * C R) /[1+\operatorname{EXP}(a+b * C R)] \tag{2}
\end{equation*}
$$

where

```
P(AW) = probability the attacker wins,
CR = casualty ratio from equation 1,
a = logistic regression intercept,
b = logistic regression slope.
```


## Task 3: Determining Neural Network Structure

Artificial neural networks (ANN) are learning systems which attempt to simulate the process of the human nervous system with the hope of capturing some part of the power of these biological systems. A typical ANN consists of one input layer, one or more hidden or middle layers, and one output layer. Each layer consists of many highly interconnected processing elements or artificial neurons, which mimic the neurons in the nervous system. Each neuron
or processing element has multiple paths carrying input signals, and one output path. These are analogous to the dendrites and axons of a human neuron. The signal that travels along each path has a specific weight that represents the outcome of a learning process, similar to the process in human synapses. All input signals are weighted and summed before producing an output signal. This output signal is generated by modifying the weighted sum by an a priori transfer function. The weights on the connections are modified by a learning rule and the procedure is repeated until an acceptable level of performance is achieved. This learning rule is critical in defining how the weights are changed in response to the input-output signal pair (Caudill and Butler 1992; Raghaven and Kanal 1992; Weiss and Kulikowski 1991; Nelson and Illingworth 1990; Sung and Johnson 1990).

There are two steps in designing any ANN. Step one is training. Step two is testing the trained model. There are two types of training. One is supervised, and the other is unsupervised. Supervised training is where the neural network is presented sets of input and desired output pairs for each treatment case. In this research, inputs are variables such as casualty ratio, and the output is whether or not the attacker won. An ANN is presented a series of such battles with their known outputs.

Unsupervised training is performed by presenting the input signal pattern only. The output is not presented.

When an ANN is trained, it is ready to be tested. This is where the network uses learned responses about the input data to predict an output. The test data consists of different inputs, such as battles, that the network has never seen.

The first step in using an ANN begins with defining the architecture of the network. The input layer consists of a processing element or node for each input variable. The output node for this study is whether the attacker won or lost.

All layers are fully connected to each adjacent layer. That is, all input layer nodes are connected to all nodes in the hidden layer. All hidden layer nodes are fully connected to the single output node.

The issue with any ANN is how many hidden layers to use and how many nodes are needed for each of those hidden layers. The number of hidden layers is determined by trial and error. The number of nodes in each hidden layer can be determined by the heuristic rule of de Villiers and Barnard (1992):

$$
\begin{equation*}
N=\sqrt{(I * O)} \tag{3}
\end{equation*}
$$

where

$$
\begin{aligned}
& N=\text { number of hidden nodes per layer, } \\
& I=\text { number of input nodes, and } \\
& O=\text { number of output nodes. }
\end{aligned}
$$

Table 3 shows Paradigm I experiments (hard, casualty input data) and forms the basis of development of the performance criteria for analysis. The values for LRA(I), for example would be the percentage of accurate predictions of the winners of the combat situations for logistic regression, for Data Set A, for Paradigm I. Similarly, percentage values would fill the remainder of the table. A similar set of experimental designs is shown for Paradigm II data in Table 4.

Each of the 12 experiments from Tables 3 and 4 is analyzed for all three data sets. Each data set is further analyzed with the two additional categories of perfect and imperfect data. Thus, there is a total of 24 experiments for the study as shown in Table 5, with 6 experimental cells for perfect data and 6 for imperfect data for each model.

## Task 5: Analvsis of Experimental Results

The results of the LR and ANN models with the perfect and imperfect data sets $A, B$ and $C$ are analyzed according to percentage of accurate predictions. The key performance statistic for comparing LR and ANN for perfect and imperfect data is the percentage of correct predictions, to be displayed in Tables 6, 7 and 8. However, before final interpretation of the percentage of accurate predictions is

| Data Set | Regression Model (LR) | Neural Network Model (BNN) |
| :---: | :---: | :---: |
| A | LRA (I) | BNNA (I) |
| B | LRB(I) | BNNB(I) |
| C | LRC (I) | BNNC (I) |

Table 3. Experiments for Paradigm I.

| Data Set | Regression Model (LR) | Neural Network Model (BNN) |
| :---: | :---: | :---: |
| A | LRA(II) | BNNA(II) |
| B | LRB(II) | BNNB(II) |
| C | LRC(II) | $\operatorname{BNNC}(I I)$ |

Table 4. Experiments for Paradigm II.

| Data Set | Regression Model (LR) | Neural Network Model (BNN) |
| :---: | :---: | :---: |
| A | LRA (I) | BNNA (I) |
| B | LRB (I) | BNNB (I) |
| C | LRC (I) | BNNC (I) |
| A | LRA (II) | BNNA (II) |
| B | LRB (II) | BNNB (II) |
| C |  | BNNC (II) |

Table 5. Summary of Experiment Design.

| Perfect versus Imperfect |  |
| :---: | :---: | :---: |
| LRA(II)P : LRA(II)IP |  |
| LRB(II)P $:$ | LRB(II)IP |
| LRC(II)P $:$ | LRC(II)IP |

## Table 6. Evaluation Chart for Satisfying the Problem Statement.

| Perfect versus Imperfect |  |
| :---: | :---: |
| LRA(II) P | LRA (II) IP |
| LRB(II) P | LRB (II) IP |
| LRC(II) P | LRC (II) IP |
| BNNA (II) P | ENNA (II) IP |
| $\operatorname{BNNB}$ (II) P | BNNB (II) IP |
| BNNC (II) P | BNNC (II) IP |

Table 7. Evaluation Chart for Satisfying Hypothesis One.


Table 8. Evaluation Chart for Satisfying Hypothesis Two.
complete, the statistical significance of those results will be determined using the Chi-Square test.

The criteria used to determine whether or not a military battle or conflict was won by the attacking force or the defending force were selected by historians, and are part of the data values in the database. Their judgments are accepted and are not part of these experiments. However, Dupuy is recognized as the authoritative source for the decision criteria used to determine winners within the database used for these experiments, having evaluated each battle and war for more than 40 years (Davis 1995). Also, the winner value was verified by an independent historical review conducted by the Concepts Analysis Agency, over a 10year period (Helmbold 1987). Therefore, for these experiments, no a priori judgments are made about winners or losers. However, the statistical and neural network models used in these experiments both use a threshold value of 0.5 to determine whether or not the dependent variable should be posted as a winner for the attacker or defender. That is, if the dependent variable is calculated as a value that is $\geq$ 0.5 , the battle is considered a win for the attacker. If the variable is calculated as $<0.5$, the battle is considered a win for the defender.

If any battle outcome is changed in any future review of this database, the results of these experiments would be suspect, requiring all models to be redeveloped. Likewise, if any new battles were added to this database, of if any
new weapon system effects were added to an existing battle, the results of these experiments would again be suspect, requiring all models to be redeveloped. Any such change in input or output variable values changes the causal relation between those variables.

## Task 6: Evaluation and Validation

Evaluation is based on determining whether the problem statement is addressed and whether the tests of hypotheses produce useful results. The models are compared for application to Paradigm I and Paradigm II, and to perfect. and imperfect stratified data sets. The results are validated by Chi-Square tests of significance at the alpha $=.05$ confidence level.

## Respurce Needs for the Experiments

The use of a neural network shell, Neuralystw, was used for the actual development. The database was created using Excel ${ }^{\text {M }}$ spreadsheet software. The computer hardware was a Macintosh system.

For the logistic regression, SAs ${ }^{\text {m }}$ was used for actual development. The SAS database was created from the same Excel ${ }^{\text {ma }}$ spreadsheet as used for the neural network analysis. The computer hardware was a Sun Sparc system.

## CHAPTER 4

EXPERIMENTATION

The first step in the experimentation process was data modeling, which examined the data elements and values within the database. The second step analyzed the data through the LR method. The third step analyzed the data through the BNN method.

There were five tasks required for the data modeling in this study. They involved the development and coding of the data as required by the LR model: data taxonomy, ratio scales, nominal scales, ordinal scales and interval scales. The degree of detail needed in the data modeling process for LR is not needed for neural networks, which is one of the differences between the BNN and LR methods.

## Data Taxonomy

The taxonomy begins by understanding that this database involves information that spans approximately 400 years of combat situations, and is described by 660 of these situations. The database was developed by Army historians with each of the more than 27,000 data items analyzed and
verified over a 10 -year period of time by the U.S. Army Concepts Analysis Agency. Thus, it is one of the authoritative sources of unclassified data on military conflicts.

The data taxonomy classifies data into four types: environment, doctrine and operations, force description, and human factors, which is shown in Table 9, along with the different data variable names. The environmental classification is for data that represent different characteristics or features of the terrain, natural atmospheric conditions, and any man-made conditions. The force description classification is for data that represent the organization of military units, such as personnel and equipment, that reflect command and control relationships and associated performance measures. The doctrine and operations classification is for data that describe the military tactics or doctrine (i.e., how to fight) used by the military forces. Human factors data represent the interaction of the personnel with the environment, the equipment within the force description, and the tactics and doctrine.

For purposes of clarification of language, throughout the remainder of this document, this military combat data will continue to be referred to as "the data," whether the intent is to describe the whole database of 660 combat situations, or any subset of the 660 , or other characteristics of the data.

| Data Classification | Data Variable Name |
| :---: | :---: |
| Environment | - Terrain <br> - Weather <br> - 1st Width of Front |
| Force Description | - Total Personnel Strength <br> - Initial Personnel Strength <br> - Horse Cavalry <br> - Total Tanks <br> - Lite Tanks <br> - Artillery Tubes <br> - Close Air Support <br> - Win or Loss <br> - Casualties |
| Doctrine and Operations | - Defensive Posture <br> - Defender's Primary Defense <br> - Attacker's Primary Tactics <br> - Defender's Primary Tactics |

Table 9. Taxonomy for Military Conflict Database.

Table 9 Continued.

| Data Classification | Data Variable Name |
| :---: | :---: |
| Human Factors | - Relative Combat Effectiveness <br> - Relative Leadership Advantage <br> - Relative Training Advantage <br> - Relative Morale Advantage <br> - Relative Logistics Advantage <br> - Relative Momentum Advantage <br> - Relative Intelligence Advantage <br> - Relative Technology Advantage <br> - Relative Initiative Advantage <br> - Attacker's Surprise |

## Pata Coding: Ratio Scale

Data coding involves examining the original 41 data variables to determine how best to define and measure variables. The original data was coded as values belonging to either the attacker or defender force. In some cases, however, there are data pairs, such as the 1 st width of the front for the attacker and the 1st width of the front for the defender, that can be combined to form a ratio variable by dividing the attacker value by that of the defender value. The result is a new variable that is coded to represent both the attacker and defender. A survey of the data indicated 20 variables that could be coded as ratios, creating 10 new ratio variables. These 10 ratio variables are listed in Table 10.

The 10 variables in Table 10 are scaled as positive ratio values, with high scores indicating attacker advantage, and low scores indicating defender advantage. The heuristic rule or equation followed was:
INPUT \#1 = ATKIST/DEF1ST
where INPUT \#1 = attacker and defender 1st Width of Front,

ATK1ST $=$ the attacker 1st Width of Front, and

DEF1ST $=$ the defender 1st Width of Front.

- 1st Width of Front of the Attacker and Defender
- Total Personnel Strength of the Attacker and Defender
- Initial Personnel Strength of the Attacker and Defender
- Horse Cavalry of the Attacker and Defender
- Total Tanks of the Attacker and Defender
- Lite Tanks of the Attacker and Defender
- Main Battle Tanks of the Attacker and Defender
- Artillery Tubes of the Attacker and Defender
- Close Air Support of the Attacker and Defender - Casualties of the Attacker and Defender

Table 10. Data Variables Coded as Ratios.

Some original data values have a value of $\mathbf{- 1}$. For the 1st Width of Front, this indicates that it was an unknown data value. Thus, creating ratios with equation 4, when one or both of the numerator or denominator could be -1 , needed further analysis. Therefore, the equation 4 for the final coding of the 1st Width of Front data element is:

> IF ATK1ST >0 AND DEFIST >0, then
> INPUT \#I = ATK1ST/DEFIST, or

IF DEF1ST $=-1$, then
INPUT \#1 $=0$,
DEF1ST $=-1=$ an unknown value,
where DEF1ST $=-1=$ an unknown value,
and,
IF ATK1ST $=-1$ AND DEF1ST $=-1$, INPUT \#1 = -1,
where $\quad$ ATK1ST $=$ DEF1ST $=-1=$ both are unknown.

With this specific variable, there were no cases where both the attacker 1st Width of Front was unknown and the defender known. A sample of the data for the 1st Width of Front variable, labeled as Input \#1, with nine of the treatment cases from the total database, is shown in Table 11. This table shows the two original data values for the attacker (i.e., ATK1ST) and defender (i.e., DEF1ST), and the ratio of ATK1ST to DEFIST, which produced Input \#1.

| INPUT \#1 | ATK1ST | DEF1ST |
| :---: | :---: | :---: |
| 1ST | 1ST | 1ST |
| FIDTH | WIDTH OF | WIDTH OF |
| FRONT | FRONT | FRONT |
| ATK/DEF RATIO | ATTACKER (ATK) | DEFENDER (DEF) |
|  |  |  |
| 0.916667 | 4.4 | 4.8 |
| 1.000000 | 0.9 | 0.9 |
| 1.000000 | 1.5 | 1.5 |
| 1.321429 | 3.7 | 2.8 |
| -1.000000 | -1.0 | -1.0 |
| -1.000000 | -1.0 | -1.0 |
| 0.888889 | 3.2 | 3.6 |
| 0.000000 | 39.0 | -1.0 |
| 0.851852 | 2.3 | 2.7 |

Table 11. Data Coding for 1st Width of Front.

For the 660 treatment cases of the 1st Width of Front variable, there are 13 cases of the attacker and defender having -1 values, and 30 cases of 0 values. Thus, approximately $6.5 \%$ of the 1st Width of Front data can be considered as imperfect. All other positive values are defined as perfect data.

In general, when using LR, as implemented by $S A S A^{\mathbf{N a}}$, an imperfect data value causes the entire treatment case to be eliminated during creation of the logistic regression model. This feature of LR occurs for the ratio coded data and any other coded data. Thus, in Table 11, this means that, of these nine sample treatment cases, the two data values under Input \#1 with data values of -1 would be eliminated. The values of 0 , although also imperfect, would be used by $L R$, since zero is a positive value.

When creating the perfect data sets, the 0 and -1 values for this data variable, shown in Table 11, were both transformed. This process involved calculating the mean from the positive variable data and replacing the values of -1 and 0 with this mean value. This method places the unknown case values within the range of values of the total population. In the example of the variable 1st Width of Front, where only $6.5 \%$ of the values are unknown, the impact on the regression should be minimal. This process of replacing the unknowns with the mean value of a variable became the general rule for creating perfect data. This does not, however, preclude the use of the earlier imperfect
coded data set in neural network modeling. In fact, one of the strengths of the neural network methodology is that it processes such data, whereas regression models would not process much of that data.

A similar coding process was completed on the other data variables shown in Table 10. The final data coding for the ratio variables is shown in Table 12. Appendix A contains the complete listing of these and all coded variables.

Table 12 indicates the degree of complexity of coding the data into useful ratio scales. The degree of uncertainty or unknown values within these 10 ratio variables is approximately 38\%. Of course, a significant number of the unknowns come from variables that were not recorded or relevant for the entire 400 years - for example, horse cavalry, tanks and close air support. For these variables, decisions have to be made on their use within the statistical regression and neural network models, which is described later.

## Data Coding: Nominal Scale

There are nine data variables that have nominal or symbolic values. The data coding for the three variables of the Defender's Primary Tactics are described here.

| Variable Name | Code and Value | Name in Appendix A |
| :---: | :---: | :---: |
| - 1st Width of Front | $\begin{aligned} >1.0= & \text { Attacker } \\ & \text { Advantage } \\ <1.0= & \text { Defender } \\ & \text { Advantage } \\ -1= & \text { Uncertainty } \\ & \text { For Both } \\ 0= & \text { Defender } \\ & \text { Uncertainty } \end{aligned}$ | Input \#1 |
| - Total Personnel Strength | $>1.0=$ Attacker <br> Advantage <br> $<1.0=$ Defender <br> Advantage | Input \#7 |
| - Initial <br> Personnel Strength | >1.0 = Attacker <br> Advantage <br> $<1.0=$ Defender <br> Advantage <br> -1 = Uncertainty <br> For Both <br> 0 = Uncertainty <br> For Either | Input \#8 |
| - Horse Cavalry | >1.0 = Attacker <br> Advantage <br> $<1.0=$ Defender <br> Advantage <br> -1 = Uncertainty <br> For Both <br> 0 = Uncertainty For Either <br> -9 $=$ Information Missing For Both | Input \#9 |

Table 12. Coding Scheme for Attacker to Defender Ratio Variables.

Table 12 Continued.

| Variable Name | Code and Value | Name in Appendix A |
| :---: | :---: | :---: |
| - Total Tanks | $\begin{aligned} &>1.0= \text { Attacker } \\ & \text { Advantage } \\ & \text { Ad.0 }= \text { Defender } \\ & \text { Advantage } \\ &-1= \text { Uncertainty } \\ & \text { For Both } \\ & 0= \text { Uncertainty } \\ & \text { For Either } \\ &-9= \text { Information } \\ & \text { Missing For } \\ & \text { Both } \\ & 9= \text { Attacker } \\ & \text { Known, but } \\ & \text { Defender 0 } \\ & 0.1= \text { Attacker 0, } \\ & \text { Defender } \\ & \text { Known } \end{aligned}$ | Input \#10 |
| - Lite Tanks | Same as Input \#10 | Input \#11 |
| - Main Battle Tanks | Same as Input \#10 | Input \#12 |
| - Artillery Tubes | Same as Input \#10 | Input \#13 |
| - Close Air Support | Same as Input \#10 | Input \#14 |
| - Casualties | $\begin{aligned} >1= & \text { Defender } \\ & \text { Advantage } \\ <1= & \text { Attacker } \\ & \text { Advantage } \\ -1= & \text { Uncertainty } \\ & \text { For Both } \\ 0= & \text { Uncertainty } \\ & \text { For Either } \end{aligned}$ | Input \#30 |

Each variable has a symbolic code to describe the tactic, such as:

```
DE = feint or a holding attack
EE = single envelopment
FF = frontal attack
PP = penetration
00 = unknown
    O = also unknown
```

There are 13 such symbolic codes for all the variables that describe the defender's and attacker's tactics. An example of the amount of variation in this data for Defender's Primary Tactics \#2 is shown in Table 13.

As shown in Table 13, 63.8\% of the information on this second part of three parts of the defender's tactical plan of operations is unknown. However, the known values need to be coded into numeric values for use in the statistical regression model. Therefore, a coding method had to be devised. One method would be to code the five symbolic values as five new variables, given that 00 and 0 are combined, and where each of these variables would be a 0,1 variable. Another method would be to have a five-valued variable.

Rather than increase the number of variables, and in order to reduce the number of values to be included in the analysis, a grouping scheme was devised. It appears in Table 14, where each value is coded such that the lowest

| Symbolic <br> Value | Frequency | Percent | Cumulative <br> Frequency | Cumulative <br> Percent |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| 0 | 392 | 59.4 | 392 | 59.4 |
| 00 | 29 | 4.4 | 421 | 63.8 |
| DE | 19 | 2.9 | 440 | 66.7 |
| EE | 24 | 3.6 | 464 | 70.3 |
| FF | 195 | 29.5 | 659 | 99.8 |
| PP | 1 | 0.2 | 660 | 100.0 |

Table 13. Frequencies of Symbolic Values for Defender's Primary Tactics \#2.

| Symbolic Value | Combined <br> Frequency | Code and Value |
| :--- | :---: | :---: |
| - 0 and 00 | 421 | 0 and $00=2$ |
| FF $=1$ |  |  |
| - All Others | 195 | Others $=0$ |

Table 14. Coding Scheme for the Nominal Valued Variable Defender's Primary Tactics \#2.
frequency value is a 0 , the next is $a 1$, and so forth. A similar coding methodology was used for the other nominal or symbolic valued variables. The complete frequency tables for all nine variables is in Appendix B. The complete coding scheme for these nominal values is shown in Table 15. For the eight input variables in Table 15, approximately $24 \%$ of the coded data have unknown values. The Output \#1 is the dependent variable to be used for both the statistical regression modeling and neural network modeling. The original data was coded $+1,-1,0$, and " -9 ". The +1 symbolized the attacker force winning the conflict. The -1 indicated the attacker lost. The 0 indicated either a tie or unknown outcome; the " -9 " was the coding for this type of variable. For coding these variables into the two values of 1 or 0 , the tied conditions were treated as if the defender won. A tied condition is considered a case where the attacker did not succeed. Therefore, the advantage goes to the defender. Only about $6 \%$ of the conflicts are recorded as a tie condition. Therefore, it was assumed the values of the regression coefficients or the weights in the neural networks would be relatively unaffected if these ties were treated differently, i.e., either ignored or an attacker win (Helmbold 1987). The two cases of uncertainty were likewise coded as a defender win. This coding is supported by previous coding experience with this and similar military data (Helmbold 1987). The final coding for Output \#1 was that the +1 became the 1 , and the $0,-1$ and

| Variable Name | Code and Value | Name in Appendix A |
| :---: | :---: | :---: |
| - Terrain | $\begin{array}{r} \text { RMO }=2 \\ \text { GMO }=1 \\ \text { Others }=0 \end{array}$ | Input \#4 |
| - Weather | $\begin{aligned} \text { DSTT } & =3 \\ \text { DSHT } & =2 \\ \text { WLTT } & =1 \\ \text { Others } & =0 \end{aligned}$ | Input \#5 |
| - Attacker's Primary Tactics \#1 | $\begin{aligned} \mathrm{FF} & =1 \\ \text { Others } & =0 \end{aligned}$ | Input \#24 |
| - Attacker's Primary Tactics \#2 | $\begin{aligned} 0 \text { and } 00 & =2 \\ E E & =1 \\ \text { Others } & =0 \end{aligned}$ | Input \#25 |
| - Attacker's Primary Tactics \#3 | $\begin{array}{r} 0 \text { and } 00=1 \\ \text { Others }=0 \end{array}$ | Input \#26 |
| - Defender's <br> Primary Tactics \#1 | $\begin{aligned} D D & =2 \\ D O & =1 \\ \text { Others } & =0 \end{aligned}$ | Input \#27 |
| - Defender's <br> Primary Tactics \#2 | $\begin{aligned} 0 \text { and } 00 & =2 \\ F F & =1 \\ \text { Others } & =0 \end{aligned}$ | Input \#28 |
| - Defender's <br> Primary Tactics \#3 | $\begin{array}{r} 0 \text { and } 00=1 \\ \text { Others }=0 \end{array}$ | Input \#29 |
| - Win or Loss for Attacker | $\begin{aligned} \text { Win } & =1 \\ \text { Loss } & =0 \end{aligned}$ | Output \#1 |

Table 15. Coding Scheme for Nominal Valued Variables.
-9 became the 0 . Thus, we have only two conditions, of attacker wins (value 1) and attacker loses (value 0).

## Data Coding: Ordinal Scale

There are 10 variables measured on an ordinal scale. The values range from +4 to -4 , as well as the -9 value for unknown information. This scale is depicted in Table 16. Like the ratio scaled variables, these ordinal variables also contain linked information about the attacker and defender, and thus are expected to be strong variables to help explain the complexity of combat. The variables that use this ordinal scale are shown in Table 17.

There are approximately $5 \%$ of these 10 ordinal variables that contain -9 values indicating unknown information about the variable. From a statistical viewpoint, this small percentage of uncertainty in this subset of the total population is considered not to have a significant impact on the prediction models. Therefore, for ease of computation, the -9 values were recoded as 0 's, which are the midpoint values in the scale range.

## Data Codino: Interval Scale

It was decided to code two variables on interval scales, although the logic of this could be debated. These variables are considered inherently different from the other tactical variables coded on nominal scales. These are

```
- +4 = Attacker is Very Strongly Favored
- +3 = Attacker is Strongly Favored
- +2 = Attacker is Favored
- +1 = Attacker is Somewhat Favored
- 0 = Neither Attacker nor Defender is Favored
- -1 = Defender is Somewhat Favored
- -2 = Defender is Favored
- -3 = Defender is Strongly Favored
- -4 = Defender is Very Strongly Favored
- -9 = Unknown Information
```

Table 16. Coding Scheme for Ordinal Valued Variables.


Table 17. Variable Names for Ordinal Variables.
the Defensive Posture Type (number of types) and the Defender's Primary Defensive Posture Type. Table 18 shows the coding scheme for these variables.

For the Defensive Posture TYpe, all values are positive. The uncertain or unknown information is coded as a 3, which assumes more than one posture type. Similarly, for Defender's Primary Defensive Posture Type, the codes are positive, but with a value of -1 for uncertain or missing information. The number of these interval variables that have unknown values was approximately 0.1\%. Therefore, it was assumed that the value can be ignored or replaced with the value of the population mean, without introducing significant error.

## Treatment Case Data Sets

Paradigms I and II prescribe the kinds of treatment case data sets to analyze. Both paradigms use the Output \#1 variable, the attacker win or loss of the military conflict, as the dependent variable. However, Paradigm I uses Input \#30, the attacker and defender casualty ratio, as the input or independent variable. Paradigm II uses the range of input variables \#1 to \#29.

Paradigm I assumes that predicting combat winners can be based on casualty ratios, the outcome of the combat situation. This method ignores all other factors that

| Variable Name | Code and Value | Name in Appendix A |
| :---: | :---: | :---: |
| - Defensive Posture Type | $0=1$ Defensive <br> Posture Type <br> $1=2$ Distinct <br> Defensive <br> Posture Types <br> $2=>2$ Averaged <br> Defensive <br> Posture Types <br> 3 = >1 Defensive <br> Posture Type <br> With Unknowns | Input \#2 |
| - Defender's <br> Primary <br> Defensive <br> Posture Type | $0=$ Hasty Defense <br> 1 = Prepared Defense <br> 2 = Fortified Defense <br> 3 = Delaying Action <br> $4=$ Withdrawal <br> $-1=$ Unknown | Input \#3 |

Table 18. Coding Scheme for Interval Valued Variables.
preceded the conflict or that took place during the process of battle. Thus, Paradigm I attempts to combine the effects of all input factors, along with their noise and uncertainty, into one outcome variable.

Paradigm II assumes that predicting combat winners can be based on the input conditions of the combat situations. This method takes into account factors leading to the operational planning before combat begins and integrates these factors, along with their inherent noise and uncertainty, into a hypercomplex relationship that reflects the social structure and human aspects of military combat. Whereas Paradigm I forces the complexity of combat into one variable, Paradigm II uses 29 variables. These are shown in Table 19.

| Paradigm | Output Variable | Input Variables |
| :---: | :---: | :---: |
| I | Output \#1 | Input \#30 |
| II | Output \#1 | Input \#1 to 29 |

Table 19. Variables for Two Paradigms.

Besides the data sets that support the two paradigms I and II, data sets are also categorized as perfect or imperfect. The perfect data, as described in earlier
sections of this document, are those data elements that have been modified to filter the uncertain or unknown values within the database. This modification replaces uncertain or unknown values with mean or midpoint values. The imperfect category is for umodified data. That is, they contain the indicators of uncertainty, such as -1 or -9.

Additionally, according to historians and analysts of similar historical combat data, there is a natural break point within this 400 years of combat. This occurs around the 1812-1815 time period. This is the time when technology began changing the doctrine and operations of military combat (Arquilla 1992). This led to a stratification of the 660 combat situations into three subsets. The Data Set A, for pre-1812 conflicts, contains 149 treatment cases. The Data Set B, for post-1812 conflicts, contains 511 treatment cases. The Data set C, for 1500-1982, contains all 660 treatment cases. The overall stratification of the database is presented in Tables 20 and 21.

## Summary

Data modeling is a first step in preparing a database for the application of analysis tools. The process of using statistical regression methods forces the analyst to understand each data element and its possible impact on the results from such analysis. This is due to the limitations of statistical regression when applied to highly variable,

| Paradigm | Perfect | Imperfect |
| :---: | :---: | :---: |
| I | $\% L R$ and $B N N$ | $\%$ LR and $B N N$ |
| II | $\%$ LR and $B N N$ | $\%$ LR and $B N N$ |

Table 20. Contingency Table for Comparing Paradigms $I$ and II with Perfect and Imperfect Data.

| Data Set | Perfect | Imperfect |
| :---: | :---: | :---: |
| A | $\%$ LR and BNN | $\%$ LR and BNN |
| B | $\%$ LR and BNN | $\%$ IR and BNN |
| C | $\%$ LR and BNN | $\%$ LR and BNN |

Table 21. Contingency Table for Comparing Data Sets A, B, and C with Perfect and Imperfect Data.
suspect or imperfect data. On the other hand, the neural network modeling approach does not need such detailed data modeling.

## Statistical Modeling

The statistical model for this research is logistic regression. LR or logit is very similar to other regression methods. However, logit or LR can be used to generate a binary dependent variable (Y). Thus, from the perspective of combat where the predicted output is the attacking force's win (1) or loss (0), LR seems appropriate. The classic linear-logistic model to predict some binary or dichotomous variable, such as a combat win or loss, is:

$$
\begin{equation*}
P(Y=Y 1)=1 /\left[1+E X P\left[-\left(B_{0}+B_{1} X_{1}+B_{2} X_{2}+\ldots+B_{n} X_{n}\right)\right]\right. \tag{6}
\end{equation*}
$$

where

$$
\begin{align*}
& Y_{1}=\text { the predicted output value of } 1, \\
& X_{i}=\text { the input variables, for } i=1 \ldots n, \\
& B_{j}=\text { the logistic regression coefficients, } \\
& \text { for } j=0 \ldots n, \text { and, } \\
& P(Y=Y 2)=1-P(Y=Y 1) \tag{7}
\end{align*}
$$

where $\mathrm{y} 2=$ the predicted output value of 0 .

This model is usually transformed into the logit equation, which calculates log-odds ratio as follows:

$$
\begin{equation*}
\text { Loge }\left[P\left(Y=Y_{1}\right) / P(Y=Y 2)\right]=B_{0}+B_{1} X_{1}+B_{2} X_{2}+\ldots+B_{n} X_{n} \tag{8}
\end{equation*}
$$

The above equations 6, 7 and 8 are described in statistics textbooks (Hosmer and Lemeshow 1989; Aldrich and Nelson 1984). And, while other methods such as multiple regression and discriminant analysis have been used for binary or dichotomous output, logistic regression is the one that seems best for such output. The other methods suffer from assumptions of normality of the data. This set of combat data is known for its non-normality (McQuie 1988). As such, the logistic regression handles data that is non-normal, and that has non-constant variance. Therefore, there are several reasons why logistic regression was chosen as the model to compare with neural networks. The rationale can also be seen when a logistic function is graphed, which shows an elongated $S$ curve shape, or sigmoid, as shown in Figure 1, where the calculated dependent variable ( $Y$ ) is between 1 (a win) and 0 (a loss).

As seen in Figure 1, the calculated value, $Y$, tends to increase as the values of the dependent variables, $X_{i}$, increase. Whether there is one variable or 29 variables, the $X_{i}$ describe the conditions for an attacker or defender win. For the left side of the curve in Figure 1, the defender wins with certainty. As we move to the right,


Figure 1. The Logistic Function.
uncertainty is introduced with respect to whether the defender wins or not, but the probability is still higher for a defender win. When the threshold value is exceeded, the odds shift to the attacker and increase as we move further to the right until the curve levels off, at which point the attacker wins with certainty (Arquilla 1992; Aldrich and Nelson 1984).

The logistic model used in this research was developed with SAS ${ }^{\mathbf{N}}$, which places certain demands on the data. For example, logit produces dichotomous dependent variable values of 1 and 0 . For the independent variables, logit is structured to accept ratio, ordinal, nominal and interval data. Thus, the coding scheme presented earlier was needed to support logit.

Following a review of the data sets for acceptable numeric values for logit, the sequence was to analyze the simpler data set for Paradigm I, followed by the larger, more complex data set for Paradigm II.

The statistical modeling using logit, then, began with Paradigm I, with the perfect database, where there is only one input variable, the ratio of casualties. Of the 660 combat situations, only five cases had any uncertainty. Therefore, the amount of imperfect data contributed approximately $0.7 \%$ to the overall population. As it turned out, the results for the perfect and imperfect databases were identical. These are presented in Table 22. The intercepts and coefficients were -0.0217, -4.8776; -0.3151, $-2.6418 ; 0.5890,-0.5372$. While it does not appear that the logit method for analyzing this single variable contributes

| Data Sets | Logit |
| :---: | :---: |
| A | $55 \%$ |
| B | $55 \%$ |
| C | $55 \%$ |

Table 22. Logit Results for Paradigm I.
much to meaningful understanding of the combat data, further statistical analysis was conducted and is presented in Chapter 5. The equivalence of the results for the three
data sets is interesting from a theoretical standpoint, and could be part of a future analysis for different stratifications of the database.

Helmbold (1987) reported logistic regression results of $72 \%$ accuracy from what would be Data Set B. However, he excluded a significant number of battles that were judged by him to have uncertain conclusions as to which side won.

The next step was to examine Paradigm II data. The experiment began with Paradigm II for Data Set A, using the imperfect data model. Data Set A contained the 149 cases from 1600 to 1812. The imperfect or unknown data values were all used. The logit eliminated variables that it determined were redundant, or for which the values indicated their irrelevance. The variables that were affected were the ratio variables of Total Tanks, Lite Tanks, Main Battle Tanks, and Close Air Support, and the nominal variable Defender's Primary Tactics \#1. This automatic transformation of the variables indicates the power of the logit not to be placed in a position of processing data that should not be there. It can be argued, for example, that the variables representing tanks and close air support were not available during the time 1600-1812. However, this a priori knowledge was not used to alter the data in this experiment. These out-of-time variables were part of the population of variables that contributed to the imperfect nature of the data model.

The results of the logit are shown in Table 23. The prediction accuracy for the values in Table 23 are $94.6 \%$ concordant, $3.1 \%$ discordant, $2.3 \%$ tied. The most influential variables from the Chi-Square, at the alpha $=$ 0.05 level of significance, appear to be Relative Leadership Advantage (Input \#16), Relative Combat Effectiveness Advantage (Input \#15), and Relative Intelligence Advantage (Input \#21). However, care must be taken in considering the results from the Chi-Square with the nonlinear data, as it can be misleading. It assumes that the variables are independent and that there are no higher-order relationships of significance.

The next analysis, for Paradigm II, Data Set B, with imperfect data, is of the post-1812 data, containing 511 cases. The results are shown at Table 24. The prediction accuracies for the values in Table 24 are $83.5 \%$ concordant, 16.4\% discordant, and $0.1 \%$ tied. Chi-Square indicates that the key variables could be Attacker Surprise Over Posture Awareness (Input \#6), Relative Leadership (Input \#16), Relative Morale (Input \#18), and Relative Technology (Input \#22).

For Paradigm II, Data Set C, with imperfect data, the results for examining the entire database with logit are shown in Table 25. The prediction accuracies are 85.4\% concordant, $14.5 \%$ discordant, and $0.1 \%$ tied. The key variables estimated from Chi-Square appear to be Attacker Surprise Over Posture Awareness (Input \#6), Artillery Tubes

| Variable | DF | Para. <br> Estimate | Standard Error | $\begin{gathered} \text { Wald } \\ \text { Chi-Sq. } \end{gathered}$ | $\begin{gathered} \text { Pr>Chi- } \\ \text { Sq. } \end{gathered}$ | std. Estimate | odds Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 1 | 6.7768 | 5.4515 | 1.5453 | 0.2138 |  | 877.280 |
| Input | 1 | 0.5885 | 1.0351 | 0.3232 | 0.5697 | 0.24925 | 1.801 |
| Imput \#2 | 1 | 0.1973 | 0.9450 | 0.0436 | 0.8346 | 0.04794 | 1.218 |
| Imput | 1 | 0.3982 | 0.7539 | 0.2791 | 0.5973 | 0.10069 | 1.489 |
| Input ${ }^{\text {\% }}$ | 1 | -0.0310 | 0.5444 | 0.0032 | 0.9546 | -0.01236 | 0.969 |
| Imput \#5 | 1 | -0.5650 | 0.3475 | 2.6432 | 0.1040 | -0.33779 | 0.568 |
| Imput \% | 1 | -0.1813 | 0.6051 | 0.0898 | 0.7645 | -0.09339 | 0.834 |
| Input \#7 | 1 | -1.7401 | 1.1912 | 2.1340 | 0.1441 | -1.44819 | 0.175 |
| Imput \#8 | 1 | 1.2021 | 1.1706 | 1.0546 | 0.3045 | 1.30805 | 3.327 |
| Imput | 1 | -0.0608 | 0.1448 | 0.1762 | 0.6747 | -0.08079 | 0.941 |
| Input \#10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Input \#11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Input 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Input ${ }_{\text {\% }} 13$ | 1 | 0.0301 | 0.2109 | 0.0204 | 0.8864 | 0.03224 | 1.031 |
| Input \#14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Input \#15 | 1 | -2.9123 | 1.4217 | 4.1963 | 0.0405 | -1.08284 | 0.054 |
| Input \#16 | 1 | -2.7806 | 0.6125 | 20.6107 | 0.0001 | -1.50015 | 0.062 |
| Imput ${ }^{17}$ | 1 | -0.4740 | 1.1651 | 0.1655 | 0.6841 | -0.16217 | 0.622 |
| Imput \#18 | 1 | -3.9377 | 1.7723 | 4.9365 | 0.0263 | -0.63523 | 0.019 |
| Input \#19 | 1 | -6.8876 | 5.8981 | 1.3637 | 0.2429 | -0.62428 | 0.001 |
| Input \#20 | 1 | -1.5413 | 1.3971 | 1.2172 | 0.2699 | -0.29491 | 0.214 |
| Input 21 | 1 | -2.0484 | 0.9027 | 5.1495 | 0.0233 | -0.73118 | 0.129 |
| Imput \#22 | 1 | -41.8068 | 5.3246 E 9 | 0.0000 | 1.0000 | -3.27794 | 0.000 |
| Input \#23 | 1 | -1.6584 | 1.8462 | 1.0254 | 0.2015 | -0.31575 | 0.2351 |
| Imput \#24 | 1 | -0.8850 | 1.9355 | 0.2091 | 0.6475 | -0.17688 | 0.413 |
| Input ${ }^{\text {W25 }}$ | 1 | 0.3315 | 0.9585 | 0.1196 | 0.7294 | 0.14193 | 1.393 |
| Input \#26 | 1 | -1.3224 | 1.6461 | 0.6454 | 0.4218 | -0.23841 | 0.266 |
| Input \#27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Imput 信 | 1 | 0.1712 | 0.7782 | 0.0484 | 0.8259 | 0.06319 | 1.187 |
| Input *29 | 1 | -0.6224 | 1.4557 | 0.1828 | 0.6690 | -0.09002 | 0.537 |

Table 23. Logit Analysis for Paradigm II, for Data Set A with Imperfect Data.

| Variable | DF | Para. Estimate | Standard Error | $\begin{aligned} & \text { Wald } \\ & \text { Chi-Sq. } \end{aligned}$ | $\begin{gathered} \text { Pr>Chi- } \\ \text { Sq. } \end{gathered}$ | std. Estimate | Odds <br> Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 1 | -7.3811 | 4.0059 | 3.3950 | 0.0654 | - ${ }^{\circ}$ | 0.001 |
| Imput \%1 | 1 | 0.2271 | 0.4652 | 0.2384 | 0.6254 | 0.04993 | 1.255 |
| Imput \#2 | 1 | -0.0909 | 0.1740 | 0.2734 | 0.6011 | -0.04902 | 0.913 |
| Imput \#3 | 1 | -0.0870 | 0.1434 | 0.3679 | 0.5441 | -0.04509 | 0.917 |
| Imput \#4 | 1 | 0.0040 | 0.0129 | 0.0010 | 0.9752 | 0.00205 | 1.004 |
| Imput \#5 | 1 | 0.0290 | 0.0969 | 0.0896 | 0.7647 | 0.02078 | 1.029 |
| Input \#6 | 1 | -0.4322 | 0.1544 | 7.8393 | 0.0051 | -0.21299 | 0.649 |
| Input \#7 | 1 | -0.1643 | 0.1173 | 1.9618 | 0.1613 | -0.19419 | 0.848 |
| Imput \#8 | 1 | -0.0038 | 0.0867 | 0.0020 | 0.9645 | -0.00543 | 0.996 |
| Input \#9 | 1 | -0.0293 | 0.0450 | 0.4255 | 0.5142 | -0.05831 | 0.971 |
| Imput \#10 | 1 | -0.0606 | 0.0415 | 2.1272 | 0.1447 | -0.24098 | 0.941 |
| Imput \#11 | 1 | 0.0107 | 0.0224 | 0.2299 | 0.6316 | 0.04388 | 1.011 |
| Ingut \#12 | 1 | 0.0078 | 0.0447 | 0.0307 | 0.8609 | 0.02856 | 1.008 |
| Input \#13 | 1 | -0.0947 | 0.0473 | 4.0160 | 0.0451 | -0.25708 | 0.910 |
| Input \#14 | 1 | -0.0113 | 0.0178 | 0.4071 | 0.5235 | -0.04687 | 0.989 |
| Input 15 | 1 | -0.5544 | 0.3121 | 3.1545 | 0.0757 | -0.73303 | 0.574 |
| Input \#16 | 1 | -1.5211 | 0.2671 | 32.4389 | 0.0001 | -2.01132 | 0.218 |
| Input \#17 | 1 | 0.6038 | 0.3083 | 3.8353 | 0.0502 | 0.79001 | 1.829 |
| Imput \#18 | 1 | -0.7865 | 0.2535 | 9.6210 | 0.0019 | -1.05222 | 0.455 |
| Imput \#19 | 1 | -0.2158 | 0.3119 | 0.4786 | 0.4890 | -0.27854 | 0.806 |
| Input ${ }^{\text {W }} 20$ | 1 | -0.1258 | 0.2705 | 0.2161 | 0.6420 | -0.16520 | 0.882 |
| Input \#21 | 1 | -0.3524 | 0.3114 | 1.2804 | 0.2578 | -0.45621 | 0.703 |
| Input \#22 | 1 | 2.6399 | 0.4362 | 36.6231 | 0.0001 | 3.35963 | 14.012 |
| Imput 23 | 1 | -0.1305 | 0.2948 | 0.7268 | 0.7592 | -0.19820 | 0.957 |
| Imput \%24 | 1 | 0.3637 | 0.3609 | 1.0155 | 0.3136 | 0.07022 | 1.439 |
| Imput \#25 | 1 | 0.2027 | 0.1775 | 1.3042 | 0.2535 | 0.08442 | 1.225 |
| Input 鲑6 | 1 | 0.4768 | 0.3959 | 1.4507 | 0.2284 | 0.08409 | 1.611 |
| Input \%27 | 1 | 3.0852 | 1.8365 | 2.8221 | 0.0930 | 0.44670 | 21.872 |
| Input \#28 | 1 | -0.2726 | 0.2255 | 1.4610 | 0.2268 | -0.08904 | 0.761 |
| Input \#29 | 1 | -0.0108 | 0.6957 | 0.0002 | 0.9876 | -0.00112 | 0.989 |

Table 24. Logit Analysis for Paradigm II, for Data Set B with Imperfect Data.

| Variable | DF | Para． Estimate | Standard Error | $\begin{aligned} & \text { Wald } \\ & \text { Chi-Sq. } \end{aligned}$ | Pr>Chi- Sq． | std． Estimate | Odds <br> Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 1 | －8．5906 | 4.0721 | 4.4505 | 0.0349 | 0 | 0.000 |
| Imput ${ }^{\text {a }}$ | 1 | 0.4867 | 0.3116 | 2.4406 | 0.1182 | 0.13424 | 1.627 |
| Imput \＃2 | 1 | －0．1208 | 0.1683 | 0.5153 | 0.4729 | －0．05968 | 0.886 |
| Imput \＃3 | 1 | －0．0602 | 0.1372 | 0.1924 | 0.6610 | －0．03084 | 0.942 |
| Imput 4 | 1 | －0．0560 | 0.1205 | 0.2159 | 0.6422 | －0．02836 | 0.946 |
| Imput \％ 5 | 1 | －0．0552 | 0.0888 | 0.3864 | 0.5342 | －0．03905 | 0.946 |
| Imput \＃6 | 1 | －0．4448 | 0.1435 | 9.6073 | 0.0019 | －0．22152 | 0.641 |
| Imput \％7 | 1 | －0．2193 | 0.1097 | 3.9923 | 0.0457 | －0．24936 | 0.803 |
| Imput 8 | 1 | 0.0508 | 0.0814 | 0.3899 | 0.5323 | 0.06839 | 1.052 |
| Input ${ }^{\text {fr }}$ | 1 | －0．0607 | 0.0355 | 2.9188 | 0.0876 | －0．15645 | 0.941 |
| Input \＃10 | 1 | －0．0686 | 0.0415 | 2.7236 | 0.0989 | 0.27397 | 0.934 |
| Imput 蕒11 | 1 | 0.0119 | 0.0224 | 0.2825 | ． 05951 | 0.04658 | 1.012 |
| Imput \＃12 | 1 | 0.0083 | 0.0443 | 0.0357 | 0.8501 | 0.03028 | 1.008 |
| Imput 管13 | 1 | －0．1005 | 0.0442 | 5.1817 | 0.0228 | －0．24780 | 0.904 |
| Input \＃14 | 1 | －0．0111 | 0.0178 | 0.3879 | 0.5334 | －0．04466 | 0.989 |
| Input \＃15 | 1 | －0．6515 | 0.2709 | 5.7861 | 0.0162 | －0．77265 | 0.521 |
| Input 筫16 | 1 | －1．7486 | 0.2155 | 65.859 | 0.0001 | －2．10682 | 0.174 |
| Imput \＃17 | 1 | 0.4698 | 0.2645 | 3.1545 | 0.0757 | 0.54985 | 1.600 |
| Input 18 | 1 | －0．8261 | 0.2403 | 11.8234 | 0.0006 | －0．97710 | 0.438 |
| Input 19 | 1 | －0．0706 | 0.3019 | 0.0548 | 0.8150 | －0．08074 | 0.932 |
| Input 20 | 1 | －0．1089 | 0.2558 | 0.1814 | 0.6702 | －0．12687 | 0.897 |
| Imput \％21 | 1 | －0．4273 | 0.2423 | 3.1104 | 0.0778 | －0．49677 | 0.652 |
| Input ${ }^{\text {W2 }}$ | 1 | 2.9097 | 0.4054 | 51.5227 | 0.0001 | 3.28203 | 18.351 |
| Input \＃33 | 1 | －0．1240 | 0.2915 | 0.19651 | 0.6824 | －0．15840 | 0.921 |
| Imput ${ }^{\text {\％} 24}$ | 1 | 0.3867 | 0.3418 | 1.2800 | 0.2579 | 0.07523 | 1.472 |
| Imput \＃25 | 1 | 0.1465 | 0.1639 | 0.7984 | 0.3716 | 0.06139 | 1.158 |
| Input \＃26 | 1 | 0.2825 | 0.3604 | 0.6142 | 0.4332 | 0.05005 | 1.326 |
| Imput \＃27 | 1 | 4.0431 | 1.9078 | 4.4910 | 0.0341 | 0.51962 | 57.000 |
| Input ${ }^{\text {H2 }} 28$ | 1 | －0．0954 | 0.1984 | 0.2314 | 0.6305 | －0．03238 | 0.909 |
| Input \＃29 | 1 | －0．4406 | 0.5918 | 0.5544 | 0.4565 | －0．05063 | 0.644 |

Table 25．Logit Analysis for Paradigm II，for Data Set C with Imperfect Data．
(Input \#13), Relative Combat Effectiveness (Input \#15), Relative Leadership Advantage (Input \#16), Relative Morale Advantage (Input \#18), and Relative Technology Advantage (Input \#22).

Thus, for the results so far, the prediction accuracies for Paradigm II, with logit and imperfect data, are indicated in Table 26.

| Data Sets | Logit <br> Prediction |
| :---: | :---: |
| A | $94.6 \%$ |
| B | $83.5 \%$ |
| C | $85.4 \%$ |

Table 26. Logit Results for Paradigm II with Imperfect Data.

Inspection of the results indicates that the prediction of accurate battles of conflicts is highest for the time period of 1600 to 1812, for Data Set A. When Data Set B is examined, with its increase in technology, the accuracy is different from Data Set A by 11\%. For Data Set C, which combines Data Set $A$ and Data Set $B$, the results are between those of Data Set A and B, as would be expected.

The summary of the key variables during this phase of experimentation with imperfect data were:

# »Attacker Surprise Over Posture Awareness (Input \#6) <br> »Artillery Tubes (Input \#13) <br> »Relative Combat Effectiveness (Input \#15) <br> »Relative Leadership Advantage (Input \#16) <br> »Relative Morale Advantage (Input \#18) <br> »Relative Intelligence Advantage (Input \#21) <br> »Relative Technology Advantage (Input \#22) 

If the objective of this research was to optimize prediction accuracy, these variables would become the critical variables for further model development. This is not part of this research, but is offered as an indication to future researchers of the variables that might be causal for battlefield modeling efforts.

A perfect data set is one where the unknown data values were replaced with the mean values as discussed in earlier sections. For Paradigm II, Data Set $A$, with perfect data, the logit also dropped Inputs $110,11,12,14$, and 27 since logit found these five data variables to be redundant or irrelevant. Again, the five variables were Total Tanks, Lite Tanks, Main Battle Tanks, Close Air Support, and Defender's Primary Tactics \#1. The prediction accuracies are shown in Table 27 indicating $94.6 \%$ concordant, $3.1 \%$ discordant, and $2.3 \%$ tied. The significant variable was Relative Leadership Advantage (Input \#16).

For Paradigm II, Data Set B, with perfect data, the analysis is shown in Table 28. The prediction accuracies

| Variable | DF | Para． Estimate | Standard Exror | $\begin{aligned} & \text { Wald } \\ & \text { Chi-Sq. } \end{aligned}$ | $\begin{gathered} \text { Pr>Chi- } \\ \text { Sq. } \end{gathered}$ | std． Estimate | Odds <br> Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 1 | 6.0176 | 5.3176 | 1.2806 | 0.2578 | 0 | 410.601 |
| Input 解1 | 1 | －0．6483 | 1.6516 | 0.1541 | 0.6946 | －0．22057 | 0.523 |
| Input \＃2 | 1 | 0.4293 | 0.9653 | 0.1978 | 0.6565 | 0.10430 | 1.536 |
| Input \＃3 | 1 | 0.2349 | 0.7726 | 0.0925 | 0.7611 | 0.05939 | 1.265 |
| Input 言4 | 1 | －0．0161 | 0.5692 | 0.0008 | 0.9775 | －0．00640 | 0.984 |
| Imput \＃5 | 1 | －0．6177 | 0.3613 | 2.9229 | 0.0873 | －0．36929 | 0.539 |
| Imput | 1 | －0．1918 | 0.6153 | 0.0972 | 0.7553 | －0．09880 | 0.825 |
| Input \＃7 | 1 | －1．4531 | 1.2512 | 1.3488 | 0.2455 | －1．20933 | 0.234 |
| Imput 咀8 | 1 | 1.0886 | 1.2884 | 0.7140 | 0.3981 | 1.17896 | 2.970 |
| Input \％9 | 1 | 0.0013 | 0.3497 | 0.0000 | 0.9968 | 0.00090 | 1.001 |
| Input \＃13 | 1 | 0.3162 | 0.2814 | 1.2630 | 0.2611 | 0.27658 | 1.372 |
| Input \＃15 | 1 | －2．7633 | 1.4135 | 3.8218 | 0.0506 | －1．02745 | 0.063 |
| Input \＃16 | 1 | －2．9147 | 0.6700 | 18.9223 | 0.0001 | －1．57246 | 0.054 |
| Input \＃17 | 1 | －0．5255 | 1.1713 | 0.2013 | 0.6537 | －0．17978 | 0.591 |
| Input \＃18 | 1 | －3．7102 | 1.9080 | 3.7815 | 0.0518 | －0．59853 | 0.024 |
| Input \＃19 | 1 | －6．1594 | 5.3457 | 1.3276 | 0.2492 | －0．55827 | 0.002 |
| Input 20 | 1 | －2．2259 | 1.5174 | 2.1518 | 0.1424 | －0．42589 | 0.108 |
| Imput \＃21 | 1. | －2．1124 | 0.9265 | 5.1979 | 0.0226 | －0．75403 | 0.121 |
| Input \＃22 | 1 | －41．3039 | 5.33 E 9 | 0.0000 | 1.0000 | －3．23851 | 0.000 |
| Input \＃23 | 1 | －2．3681 | 1.6524 | 2.3405 | 0.1680 | －0．45038 | 0.261 |
| Imput \％ 24 | 1 | －0．3144 | 1.6982 | 0.0343 | 0.8531 | －0．06284 | 0.730 |
| Input \＃25 | 1 | 0.1305 | 0.8761 | 0.0222 | 0.8816 | 0.05588 | 1.139 |
| Input \＃26 | 1 | －0．6545 | 1.5074 | 0.1886 | 0.6641 | －0．11800 | 0.520 |
| Input \＃27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Input \＃28 | 1 | 0.1588 | 0.7665 | 0.0429 | 0.8359 | 0.05860 | 1.172 |
| Input \＃29 | 1 | －0．8176 | 1.5000 | 0.2971 | 0.5857 | －0．11826 | 0.441 |

Table 27．Logit Analysis for Paradigm II，for Data Set A with Perfect Data．

| Variable | DF | Para. Estimate | Standard Error | $\begin{aligned} & \text { Fald } \\ & \text { Chi-Sq. } \end{aligned}$ | $\begin{gathered} \text { Pr>Chi- } \\ \text { Sq. } \end{gathered}$ | std. Estimate | Odds Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 1 | -2.4195 | 2.2451 | 1.1613 | 0.2812 | 0 | 0.089 |
| Imput ${ }^{\text {mi }}$ | 1 | -0.9121 | 0.5756 | 2.5109 | 0.1131 | -0.16547 | 0.402 |
| Imput ${ }^{\text {a }}$ | 1 | -0.0130 | 0.1805 | 0.0052 | 0.9424 | -0.00702 | 0.987 |
| Imput \% 3 | 1 | 0.0088 | 0.1461 | 0.0036 | 0.9520 | 0.00451 | 1.009 |
| Irput 4 | 1 | 0.1494 | 0.1379 | 1.1735 | 0.2787 | 0.07621 | 1.161 |
| Input 5 | 1 | 0.0152 | 0.1013 | 0.0225 | 0.8806 | 0.01090 | 1.015 |
| Imput \#6 | 1 | -0.4559 | 0.1716 | 7.0569 | 0.0079 | -0.22464 | 0.634 |
| Imput \#7 | 1 | -0.2380 | 0.1539 | 2.3909 | 0.1220 | -0.28129 | 0.788 |
| Imput \#8 | 1 | 0.0046 | 0.1434 | 0.0010 | 0.9743 | 0.00539 | 1.005 |
| Imput \#9 | 1 | 0.1133 | 0.2253 | 0.2530 | 0.6150 | 0.03419 | 1.120 |
| Imput | 1 | -0.0964 | 0.0669 | 2.0740 | 0.1498 | -0.18234 | 0.908 |
| Input \#11 | 1 | -0.0376 | 0.0458 | 0.6753 | 0.4112 | -0.06813 | 0.963 |
| Input \#12 | 1 | 0.0841 | 0.0783 | 1.1546 | 0.2826 | 0.13613 | 1.088 |
| Input ${ }^{\text {\# }} 13$ | 1 | -0.0453 | 0.0536 | 0.7120 | 0.3988 | -0.11746 | 0.956 |
| Input 14 | 1 | -0.0431 | 0.0312 | 1.9103 | 0.1669 | -0.10851 | 0.958 |
| Imput 15 | 1 | -0.4605 | 0.3342 | 1.8983 | 0.1683 | -0.16896 | 0.631 |
| Input 16 | 1 | -1.9442 | 0.3221 | 36.4262 | 0.0001 | -0.71724 | 0.143 |
| Input \#17 | 1 | 0.4268 | 0.3311 | 1.6621 | 0.1973 | 0.15190 | 1.532 |
| Imput \#18 | 1 | -1.1919 | 0.2913 | 16.745 | 0.0001 | -0.39147 | 0.304 |
| Imput \#19 | 1 | -0.7498 | 0.3800 | 3.8929 | 0.0485 | -0.18725 | 0.472 |
| Input \#20 | 1 | -0.6580 | 0.3080 | 4.5643 | 0.0326 | -0.15922 | 0.518 |
| Input \#21 | 1 | -1.0847 | 0.4097 | 7.0101 | 0.0081 | -0.30408 | 0.338 |
| Input \#22 | 1 | 0.5985 | 0.5924 | 1.0206 | 0.3124 | 0.09876 | 1.819 |
| Input \#23 | 1 | -0.7025 | 0.3561 | 3.1590 | 0.0910 | -0.18056 | 0.429 |
| Input 䜌24 | 1 | 0.5008 | 0.3935 | 1.6193 | 0.2032 | 0.09670 | 1.650 |
| Imput \%25 | 1 | 0.0499 | 0.1963 | 0.0645 | 0.7996 | 0.02076 | 1.051 |
| Imput \#26 | 1 | 0.5429 | 0.4286 | 1.6046 | 0.2053 | 0.09574 | 1.721 |
| Imput ${ }^{\text {W27 }}$ | 1 | 1.1927 | 0.8036 | 2.2029 | 0.1378 | 0.17269 | 3.296 |
| Input \#28 | 1 | -0.1524 | 0.2521 | 0.3654 | 0.5455 | -0.04977 | 0.859 |
| Input \#29 | 1 | 0.0920 | 0.7847 | 0.0137 | 0.9067 | 0.00960 | 1.096 |

Table 28. Logit Analysis for Paradigm II, for Data Set B with Perfect Data.
were $87.4 \%$ concordant, $12.4 \%$ discordant, and $0.1 \%$ tied. The key variables were Attacker's Surprise Over Posture Awareness (Input \#6) and Relative Leadership Advantage (Input \#16).

For Paradigm II, Data Set C, with perfect data, the analysis is shown in Table 29. The prediction accuracies are $88.8 \%$ concordant, $11.1 \%$ discordant, and $0.1 \%$ tied. The key variables were Attacker's Surprise Over Posture Awareness (Input \#6), Relative Leadership Advantage (Input \#16), Relative Morale Advantage (Input \#18), and Relative Intelligence Advantage (Input \#21).

Thus, for Paradigm II with logit, and imperfect and perfect data, the prediction accuracies are indicated in Table 30.

The results indicate that the logistic regression performs better with perfect data versus imperfect data. That is, if the database has significant noise or uncertainty, or missing data, then logistic regression may produce a lower accuracy prediction than if the database represented a more "perfect" system.

## Neural Network Modeling

The neural network model identified for use in this experiment was backpropagation. As mentioned earlier, the backpropagation neural network is a nonlinear computational

| Variable | DF | Para. Estimate | Standard Error | $\begin{gathered} \text { Fald } \\ \text { Chi-Sq. } \end{gathered}$ | $\begin{gathered} \text { Pr>Chi- } \\ \text { Sq. } \end{gathered}$ | std. Estimate | Odds Ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 1 | -1.5962 | 2.0031 | 0.6350 | 0.4255 |  | 0.203 |
| Input \#1 | 1 | -0.4204 | 0.4402 | 0.9120 | 0.3396 | -0.09566 | 0.657 |
| Imput \#2 | 1 | 0.0139 | 0.1731 | 0.0064 | 0.936 | 0.00686 | 1.014 |
| Imput \#3 | 1 | 0.0797 | 0.1386 | 0.3310 | 0.5651 | 0.04063 | 1.083 |
| Imput 4 | 1 | 0.0629 | 0.1285 | 0.2393 | 0.6247 | 0.03184 | 1.065 |
| Imput \% 5 | 1 | -0.0688 | 0.0934 | 0.5431 | 0.4611 | -0.04869 | 0.933 |
| Imput 6 | 1 | -0.4301 | 0.1565 | 7.5528 | 0.0060 | -0.21418 | 0.659 |
| Imput \#7 | 1 | -0.2954 | 0.1514 | 3.8068 | 0.0510 | -0.33593 | 0.744 |
| Imput \% 8 | 1 | 0.1198 | 0.1377 | 0.7558 | 0.3846 | 0.14012 | 1.127 |
| Input \%9 | 1 | 0.0558 | 0.1614 | 0.1195 | 0.7295 | 0.02445 | 1.057 |
| Input \#10 | 1 | -0.0979 | 0.0667 | 2.1504 | 0.1425 | -0.16606 | 0.907 |
| Input \#11 | 1 | -0.0429 | 0.0455 | 0.8892 | 0.3457 | -0.07023 | 0.958 |
| Input \#12 | 1 | 0.0891 | 0.0772 | 1.3337 | 0.2481 | 0.12941 | 1.093 |
| Input \#13 | 1 | -0.0476 | 0.0515 | 0.8562 | 0.3548 | -0.11146 | 0.953 |
| Input \#14 | 1 | -0.0458 | 0.0326 | 1.9731 | 0.1601 | -0.10251 | 0.955 |
| Input \#15 | 1 | -0.6389 | 0.2938 | 4.7293 | 0.0297 | -0.23496 | 0.528 |
| Input 16 | 1 | -2.0460 | 0.2530 | 65.4175 | 0.0001 | -0.84883 | 0.129 |
| Imput \%17 | 1 | 0.2360 | 0.2894 | 0.6651 | 0.4148 | 0.08321 | 1.266 |
| Imput \#18 | 1 | -1.2958 | 0.2791 | 21.5611 | 0.0001 | -0.39460 | 0.274 |
| Input 19 | 1 | -0.7728 | 0.3786 | 4.1664 | 0.0412 | -0.17360 | 0.462 |
| Imput 20 | 1 | -0.6880 | 0.2920 | 5.5511 | 0.0185 | -0.16070 | 0.503 |
| Imput \#21 | 1 | -0.9985 | 0.2917 | 11.7177 | 0.0006 | -0.29908 | 0.368 |
| Imput \#22 | 1 | 0.6155 | 0.5750 | 1.1457 | 0.2845 | 0.09245 | 1.851 |
| Input \#23 | 1 | -0.6802 | 0.3584 | 3.6980 | 0.0540 | -0.25041 | 0.439 |
| Imput 24 | 1 | 0.5051 | 0.3632 | 1.9343 | 0.1643 | 0.09825 | 1.657 |
| Input \#25 | 1 | -0.0036 | 0.1766 | 0.0004 | 0.9834 | -0.00154 | 0.996 |
| Input 26 | 1 | 0.4079 | 0.3946 | 1.0686 | 0.3013 | 0.07224 | 1.504 |
| Input 27 | 1 | 0.9759 | 0.7051 | 1.9158 | 0.1663 | 0.12542 | 2.654 |
| Input 28 | 1 | 0.0391 | 0.2233 | 0.0307 | 0.8608 | 0.01328 | 1.040 |
| Input \#29 | 1 | -0.3117 | 0.6596 | 0.2233 | 0.6365 | -0.03582 | 0.732 |

Table 29. Logit Analysis for Paradigm II, for Data Set C with Perfect Data.

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| Data Sets | Logit <br> Imperfect | Logit <br> Perfect |
| :---: | :---: | :---: |
| $A$ | $94.6 \%$ | 94.6 |
| $B$ | $83.5 \%$ | 87.4 |
| $C$ | $85.4 \%$ | 88.8 |

Table 30. Logit Results for Paradigm II with Imperfect and Perfect Data.
model that uses parallel parocessing. A common metaphor is the human brain, and, like the human nervous system, nodes are connected through a weighting algorithm that determines what signal is passed from one node to another. These weights and signals are adaptive and can change based on learning as the network looks at a pattern of data many times.

Understanding the algorithmic basis of the backpropagation neural network begins at the output node (Neuralyst ${ }^{\mathbf{N a}}$ 1994; Nelson and Illingworth 1990). The output of each node for each layer is a function of the input values or weights. For example, the calculated output of the jth node begins with the following equation:

$$
\begin{equation*}
v_{j}=\sum_{i}\left(X_{i} * W_{i j}\right) \tag{9}
\end{equation*}
$$

where

$$
\begin{aligned}
U_{j}= & \text { an internal summation for the } j \text { th node, } \\
X_{i}= & \text { input from the } i \text { th node, } \\
W_{i j}= & \text { a previously established weight, such as } \\
& W_{i j}=W_{i j}^{\prime}+L R^{*} e_{j} * X_{i}
\end{aligned}
$$

where

$$
\begin{aligned}
& L R=\text { the learning rate, } \\
& e_{j}=\text { an error term for the } j \text { th node, such as } \\
& \quad e_{j}=Y_{j} *\left(1-Y_{j}\right) *\left(d_{j}-Y_{j}\right)
\end{aligned}
$$

where
$Y_{j}=$ the actual $j$ th node computed output value, $\left(1-Y_{j}\right)=$ the complement of $Y_{j}$, and
$d_{j}=$ the desired or known output value.

The summation operation, $v_{j}$, in equation 9 , is compared to a threshold value, $t_{j}$, and passed through a sigmoid activation function, $F_{t h}$, as $Y_{j}=F_{t h}\left(U_{j}+t_{j}\right)$, which is the output response for the next layer, or the final layer. An example of an artificial neuron is diagrammed in Figure 2.


Figure 2. A Model Neuron.

Once the error values are computed for the output layer and adjusted to the next layer back, the error term is modified to be $e_{j}=Y_{j} *\left(1-Y_{j}\right) * \sum\left(e_{k} *{ }^{\prime}{ }_{j k}\right)$, which replaces the difference between the desired and actual output with the sum of the error terms for each node, $k$, in the next succeeding layer.

The learning rate, $L R$, is set by the user to adjust the old weights. Finally, the weight adjustment is modified as $W_{i j}=W_{i j}^{\prime}+(1-M) * L R^{*} e_{j} * X_{i}+M^{*}\left(W_{i j}^{\prime} W^{\prime \prime}{ }_{i j}\right)$, to add a user set momentum factor, $M$, which allows some persistence of preceeding weights to the iterations of succeeding weights.

As the neural network is trained, the sum of the errors should become smaller, until it reaches a user set value, which stops the training process. The network is then ready to be applied to a test data set, using the trained weights. This testing provides some validation to the trained network.

The process followed in this section is similar to that followed in the statistical modeling section, that is, to examine the data sets for the different timeframes, for imperfect and perfect data, and for Paradigms I and II.

One difference between the neural network and logit becomes apparent as one begins to structure the network. Unlike logit, one must use trial and error approaches to determine what the network should look like. The number of input variables and the number of output variables affect this process.

The number of input nodes for Paradigm I is one, the Casualty ratio. The number of output nodes is also one. According to the heuristic rule, the number of hidden nodes then should be one. However, tests were conducted to determine if this were true. The number of hidden nodes on
the one hidden layer were tested in increments of one. Thus, network models of 1-1-1, 1-2-1, 1-3-1, 1-4-1, and 1-5-1 were run with the Paradigm I data set $A$. The results indicated that 1-3-1 provided better prediction accuracy than the other networks. In fact, the percentage of accurate predictions increased as the number of nodes were increased, to a peak at the 1-3-1, and then fell off for 1-4-1 and 1-5-1. An excursion into two hidden layers of 1-3-3-1 performed poorly. Therefore, the 1-3-1 network was chosen for use with Paradigm I.

For paradigm II, the number of input nodes is set at 29, since there are 29 input variables. The number of output nodes is similarly set as a single node. The issue again was how many hidden layer nodes to use. The heuristic of the square root of the product of the number of nodes in the input and the number in the output is a suitable answer. This came after several tests of Data Set A starting with a network structure of 29-6-1. The number of hidden nodes was changed by increments of two to determine if a better structure would work for this type of data. The tests included networks 29-4-1, 29-6-1, 29-8-1, 29-10-1, and three networks with two hidden layers, 29-6-6-1, 29-6-4-1, and 29-4-4-1. The two hidden layer network performed poorly. The single hidden layer performed best with the 29-6-1 structure as prescribed by the heuristic. Thus, the network structure used for Paradigm II was 29-6-1.

Another difference in using network methodology versus logit is that the network is trained on $80 \%$ of the data, and uses the remaining $20 \%$ for testing or validating the prediction model (Neuralyst ${ }^{\mathbf{N A}}$ 1994; Lippman 1991; Nelson and Illingworth 1990). Since 100\% of the data for logit was used to build the model, each network model also revisited the entire data set for $100 \%$ of the treatment cases. It is this $100 \%$ figure which is used to compare with that from the $100 \%$ treatment cases for logit. For purposes of clarity, the $80 \%$ and $20 \%$ rule for networks is referred to as $80 / 20$ throughout the remainder of this document.

The parameters used for the supervised network building were learning rate 0.9 , momentum 0.9 , no input noise, 0.1 training tolerance, and 0.3 testing tolerance (that is, $t_{j}$ 's are the same for all $j^{\prime} s$ ). Also, the learning rate was adaptive, meaning that the model could search for alternatives to avoid local minima traps.

The network experiment began by examining the case of the single independent variable, casualty ratio, as the basis for predicting winners of combat, that is, Paradigm I. Since there is no imperfect data for Paradigm $I$, the only data set examined was the perfect data set. There was still the timeframe stratification represented by the three data sets $A, B$, and $C$.

The results from Paradigm $I$, using Data Set $A$, for the perfect data neural network are shown in Table 31. The

| Statistics | Training Data <br> $(80 \%)$ | Testing Data <br> $(20 \%)$ | $100 \%$ Data |
| :--- | :---: | :---: | :---: |
| RMS Error | 0.5 | 27 | 149 |
| Treatments | 122 | 0 | 1 |
| Number Right | 1 | 27 | 148 |
| Number Wrong | 121 | $1 \% \%$ | $1 \%$ |
| Percent Right | $99 \%$ |  | $99 \%$ |
| Percent Wrong | 900 |  |  |
| Training |  |  |  |
| Epochs |  |  |  |

Table 31. Network Results for Paradigm I, Data Set A with Perfect Data.
results are quite different from the previous prediction values for logit, where the logit gave a prediction of 55\%.

The results from the Data Set $B$ experiment indicate something different, as shown in Table 32.

The final phase of the Paradigm I experiment is an examination of the data set for the entire 1600-1982 timeframe. The results are shown in Table 33 . While the prediction accuracy for the test set was 63\%, the training results show only one case correctly predicted.

This poor performance of the models for Paradigm $I$, with perfect data, are indicated by the high RMS errors and the low prediction accuracies. This raises questions concerning the data model, the structure of BNN, and the modeling process. The low prediction accuracies for logit analysis of Paradigm I raise similar questions about the data model.

The summary of the Paradigm I results from both logit and network modeling are given in Table 34.

For Paradigm II, Data Set A, with imperfect data, the neural network's 80/20 and 100\% results are shown in Table 35. The model was developed on $80 \%$ of the data available. Often with networks, the prediction accuracy during the training stage does not match that of the testing stage.

Another critical part in training a network is to know when the model has stabilized. This is usually seen by the root mean square (RMS) error plateauing. The goal is to drive the RMS error as close to zero as possible, although

| Statistics | Training Data <br> (80\%) | Testing Data <br> (20\%) | 100\% Data |
| :--- | :---: | :---: | :---: |
| RMS Error | 0.4 |  | 55 |
| Treatments | 416 | 57 | 292 |
| Number Right | 20 | 38 | 219 |
| Number Wrong | 396 | $60 \%$ | $57 \%$ |
| Percent Right | $5 \%$ | $40 \%$ | $43 \%$ |
| Percent Wrong | $95 \%$ |  |  |
| Training <br> Epochs | 1200 |  |  |

Table 32. Network Results for Paradigm I, Data Set B with Perfect Data.

| Statistics | Training Data <br> $(80 \%)$ | Testing Data <br> $(20 \%)$ | 100\% Data |
| :--- | :---: | :---: | :---: |
| RMS Error | 0.4 |  | 660 |
| Treatments | 549 | 111 | 399 |
| Number Right | 1 | 70 | 261 |
| Number Wrong | 548 | $63 \%$ | $60 \%$ |
| Percent Right | $1 \%$ | $37 \%$ | $40 \%$ |
| Percent Wrong | $99 \%$ |  |  |
| Training <br> Epochs | 1100 |  |  |

Table 33. Network Results for Paradigm I, Data Set C with Perfect Data.

| Data Sets | Logit | Network |
| :---: | :---: | :---: |
| A | $55 \%$ | $1 \%$ |
| B | $55 \%$ | $57 \%$ |
| C | $55 \%$ | $60 \%$ |

Table 34. Paradigm I Comparison of Prediction Accuracies for Logit and Neural Networks.

| Statistics | Training Data <br> $(80 \%)$ | Testing Data <br> $(20 \%)$ | 100\% Data |
| :--- | :---: | :---: | :---: |
| RMS Error | 0.06 | 29 | 149 |
| Treatments | 120 | 25 | 143 |
| Number Right | 118 | 4 | 6 |
| Number Wrong <br> Percent Right | 2 | $86 \%$ | $96 \%$ |
| Percent Wrong | $2 \%$ | $14 \%$ | $4 \%$ |
| Training <br> Epochs | 1500 |  |  |

Table 35. Network Results for Paradigm II, Data Set A with Imperfect Data.
for the Paradigm I cases it plateaus at a relative high level.

Table 36 is a copy of the results from the network indicating what the results look like compared to the value for each output variable. The table shows the sequence of treatment cases or military conflicts.

For Paradigm II, Data Set B, with imperfect data, the results are shown in Table 37.

For Paradigm II, Data Set $C$, with imperfect data, the results are shown in Table 38 . The results indicate that the network does well in predicting the winner of combat for Paradigm II with imperfect data.

We now have enough prediction information to take a first view of the inference capabilities of the logit and network modeling approaches. This is shown in Table 39.

The preliminary results create several pieces of information, and questions. The information is that for the two logit cases, logit seems to do slightly better at predicting when the data is more perfect than imperfect.

The range of predictions for logit with imperfect data is from $84 \%$ to $95 \%$. The prediction accuracies are relatively high given the type of data and the large number of measurement scales for the input variables. The range of predictions for the network with imperfect data was similar to the range for logit with perfect data.

The next step was to examine the network model with the perfect data set. The perfect data sets were identical to

| Battle <br> Sequence <br> Number | Output \#I <br> Attacker <br> Win (1) <br> or Loss (0) | Predicted <br> Output |
| :---: | :---: | :---: |
| 1 | 0 | -0.007938 |
| 2 | 1 | 1.0016266 |
| 3 | 1 | 1.0002502 |
| 4 | 0 | -0.049698 |
| 5 | 1 | 1.0048828 |
| 6 | 0 | 0.1070374 |

Table 36. Sample of Computed Output of Data Set A Network
with Imperfect Data.

| Statistics | Training Data <br> $(80 \%)$ | Testing Data <br> $(20 \%)$ | 100\% Data |
| :--- | :---: | :---: | :---: |
| RMS Error | 0.06 |  | 511 |
| Treatments | 417 | 81 | 454 |
| Number Right | 389 | 13 | 57 |
| Number Wrong | 30 | $87 \%$ | $89 \%$ |
| Percent Right | $93 \%$ | $13 \%$ | $11 \%$ |
| Percent Wrong | $7 \%$ |  |  |
| Training | 2900 |  |  |
| Epochs |  |  |  |

Table 37. Network Results for Paradigm II, Data Set B with Imperfect Data.

| Statistics | Training Data <br> (80\%) | Testing Data <br> $(20 \%)$ | 100\% Data |
| :--- | :---: | :---: | :---: |
| RMS Error | 0.07 |  | 660 |
| Treatments | 529 | 131 | 613 |
| Number Right | 518 | 111 | 47 |
| Number Wrong <br> Percent Right | 11 | 20 | $93 \%$ |
| Percent Wrong | $2 \%$ | $15 \%$ | $7 \%$ |
| Training <br> Epochs | 2300 |  |  |

Table 38. Network Results for Paradigm II, Data Set $C$ with Imperfect Data.

| Data Sets | Logit <br> Imperfect | Logit <br> Perfect | Network <br> Imperfect |
| :---: | :---: | :---: | :---: |
| A | $95 \%$ | $95 \%$ | $96 \%$ |
| B | $84 \%$ | $87 \%$ | $89 \%$ |
| C | $85 \%$ | $89 \%$ | $93 \%$ |

Table 39. Preliminary Comparison of Logit and Neural Network Results for Paradigm II.
those used in the logit model analysis. This was not the case for the imperfect data sets. The imperfect data for the neural network analysis used the orignal symbolic representations for the nominal variables that had been coded for logit.

For Paradigm II, Data Set A, with perfect data, the results from the network are shown in Table 40.

| Statistics | Training Data <br> (80\%) | Testing Data <br> (20\%) | 100\% Data |
| :--- | :---: | :---: | :---: |
| RMS Error | 0.04 |  | 149 |
| Treatments | 122 | 27 | 143 |
| Number Right | 122 | 21 | 6 |
| Number Wrong | 0 | 6 | $96 \%$ |
| Percent Right | $100 \%$ | $22 \%$ | $4 \%$ |
| Percent Wrong | $0 \%$ |  |  |
| Training | 1000 |  |  |
| Epochs |  |  |  |

Table 40. Network Results for Paradigm II, Data Set A with Perfect Data.

The results appear to indicate that the network provides a high percentage of correct predictions. The low RMS is another indication that the network converged. Logit also gave a high percentage for these pre-1812 cases.

For Paradigm II, Data Set B, with perfect data, the results are shown in Table 41. The results indicate that the network model did not converge as strongly as it did for the previous model. This may be a departure point for comparison of logit and neural networks. That is, for perfect data use logit, and for noisy data use neural networks. However, the experiments reported here were not set up to evaluate rigorously the two methods relative to each other. Further research is needed to verify for which data type each is more appropriate. For now, the two should be regarded as complements to each other.

For Paradigm II, Data Set C, with perfect data, the results are shown in Table 42. The results indicate that the network did not converge, as seen by the high RMS, as well as the low prediction rates for the 80/20 and 100\% cases. The relatively low prediction accuracies again raise questions about the data model. The incorporation of artificial data values (i.e., means or midpoints) for missing or uncertain data elements may confuse the neural network, if the network is trying to account for high-order relationships in the data. This is a subject for further investigation.

We can now compare the entire Paradigm II results, which are shown in Table 43.

| Statistics | Training Data <br> $(80 \%$ | Testing Data <br> $(20 \%)$ | 100\% Data |
| :--- | :---: | :---: | :---: |
| RMS Error | 0.4 |  | 511 |
| Treatments | 421 | 90 | 296 |
| Number Right | 213 | 45 | 215 |
| Number Wrong | 208 | 45 | $58 \%$ |
| Percent Right | $51 \%$ | $50 \%$ | $42 \%$ |
| Percent Wrong | $49 \%$ | $50 \%$ |  |
| Training <br> Epochs | 2000 |  |  |

Table 41. Network Results for Paradigm II, Data Set B with Perfect Data.

| Statistics | Training Data <br> $(80 \%)$ | Testing Data <br> $(20 \%)$ | 100\% Data |
| :--- | :---: | :---: | :---: |
| RMS Error | 0.3 |  | 660 |
| Treatments | 533 | 127 | 380 |
| Number Right | 338 | 68 | 280 |
| Number Wrong | 195 | 59 | $58 \%$ |
| Percent Right | $63 \%$ | $54 \%$ | $42 \%$ |
| Percent Wrong | $37 \%$ | 1800 |  |
| Training <br> Epochs |  |  |  |

Table 42. Network Results for Paradigm II, Data Set C with Perfect Data.

| Data Sets | Logit <br> Imperfect | Network <br> Imperfect | Logit <br> Perfect | Network <br> Perfect |
| :---: | :---: | :---: | :---: | :---: |
| A | $95 \%$ | $96 \%$ | $95 \%$ | $96 \%$ |
| B | $84 \%$ | $89 \%$ | $87 \%$ | $58 \%$ |
| C | $85 \%$ | $93 \%$ | $89 \%$ | $58 \%$ |

Table 43. Results of Logit and Neural Network Analysis for Paradigm II.

## CHAPTER 5 <br> ANALYSIS OF EXPERIMENTAL RESULTS

This experiment compared the predictive capabilities of traditional statistical regression and artificial neural network models for noisy, unfiltered data (i.e., imperfect) versus filtered data (i.e., perfect). The preliminary results are shown in Table 44 by percentage and frequency of correct predictions. The cell entries for Paradigm $I$ with perfect data are identical to the cell entries for Paradigm I with imperfect data, since there were no imperfect cases for Paradigm I.

The percentages could be used to propose a possible trend in the future use of logit or neural networks, depending of whether one has perfect or imperfect data. There is also reason to suggest that Paradigm II is superior to Paradigm I for modeling military combat outcomes. However, further statistical analysis of the significance of the experimental results was conducted.

## Chi-Square Tests

Analyzing the prediction results for their statistical significance provides evidence about the probability that

| Data Set Types | Analysis Model "LR" <br> Paradigm I | Analysis Model "LR" <br> Paradigm II | Analysis Model "BNN" Paradigm I | Analysis <br> Model <br> "BNN" <br> Paradigm II |
| :---: | :---: | :---: | :---: | :---: |
| Perfect Data Set "A" | 55\% (82) | 95\% (141) | 1\% (1) | 96\% (143) |
| Perfect Data Set "B" | $55 \%$ (281) | 87\% (447) | 57\% (292) | 58\% (296) |
| Perfect Data Set "C" | 55\% (363) | $89 \%$ (586) | 60\% (399) | 58\% (380) |
| Imperfect <br> Data Set "A" | 55\% (82) | 95\% (141) | 18 (1) | 96\% (143) |
| Imperfect Data Set "B" | 55\% (281) | 84\% (427) | 57\% (292) | 89\% (454) |
| Imperfect Data Set "C" | 55\% (363) | 85\% (564) | 60\% (399) | 938 (613) |

Numbers in parenthesis are frequencies of accurate predictions.

Table 44. Summary of Experimental Results.
these results could not have been produced by chance. The analysis of statistical significance used the Chi-Square test. The problem statement is tested first, followed by tests for hypothesis one, and then tests for hypothesis two. The problem for this research was that the predictive Capabilities of traditional statistical models may not be as robust when applied to noisy and incomplete data as they are when applied to clean, or filtered data, and combat data that focuses on skill-based and human factors tend to be noisy and incomplete.

To examine this problem statistically, populations of data that are perfect are compared with populations that are imperfect. Table 44 gives an indication that there may be a significant difference between perfect and imperfect data for the traditional statistical method used - LR or logit. For Data Set $A$, which consists of military engagements from 1600-1812, the table indicates that the predictions for the imperfect and perfect data type were equal. For Data Set B, which consists of engagements from 1812-1982, however, the table indicates a change in predictions for the two data types. Data Set C consists of both Data Sets A and B, and thus contains the effects of Data Set A. While the differences in the predictions for perfect and imperfect data are small, the slightly better accuracy with perfect data for Data Sets B and C, indicate that the perfect data model may provide more accurate predictions.

To test this before and after condition of the data transformation from imperfect to perfect, the Chi-Square $\left(X^{2}\right)$ statistic was used. Data Set $A$ was not used in this part of the analysis since the frequencies are equivalent for perfect and imperfect data. Similarly, the logit data for Paradigm I are equivalent for the imperfect and perfect cases, so Paradigm I results were not analyzed.

Table 45 shows a comparison of data type (i.e., perfect and imperfect) against accurate and inaccurate predictions for logit, for Paradigm II, and Data Set B. Table 46 shows the calculated $x^{2}=3.1604$ which is less than ( $<$ ) the critical $x^{2}$ value of 3.841 . This indicates that the differences in prediction accuracies caused by perfect and imperfect data are not statistically significant. But, since the computed value of $\mathrm{X}^{2}$ is different from the critical value by only 0.6806 , we conclude that the lack of statistical significance is marginal at an alpha $=0.05$ level of significance.

Next, we compute $X^{2}$ for Data Set $C$, Paradigm II, as shown in Tables 47 and 48. The computed $X^{2}=3.2678<$ 3.841, which is again less than the critical value. The difference between the computed and critical $X^{2}$ is 0.5732 which, like the case for Data Set $B$, is evidence for marginal rejection of statistical significance.

Next, we compute the $\mathrm{X}^{2}$ for hypothesis one, which examines the difference in the accuracy of the two model (i.e., LR and BNN) predictions of combat winners based on

|  | Imperfect | Perfect | $\sum$ |
| :--- | :---: | :---: | :---: |
| Accurate | 427 | 447 | 874 |
| Inaccurate | 84 | 64 | 148 |
| $\Sigma$ | 511 | 511 | 1022 |

Table 45. Comparison of Logit Results by Data Type for Paradigm II, with Data Set B.

|  | OF | EF | OF-EF | $\left(\right.$ OF-EF) ${ }^{2}$ | $(O F-E F)^{2} / E F$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Imperfect <br> Accurate | 427 | 437 | -10 | 100 | 0.2288 |
| Imperfect <br> Inaccurate | 84 | 74 | 10 | 100 | 1.3514 |
| Perfect <br> Accurate | 447 | 437 | 10 | 100 | 0.2288 |
| Perfect <br> Inaccurate | 64 | 74 | -10 | 100 | 1.3514 |
|  |  |  |  | $\Sigma$ | 3.1604 |
| df $=1$, alpha $=0.05, x^{2}$ Critical Value $=3.841$ |  |  |  |  |  |

Table 46. $X^{2}$ Chart Created from Table 45. (OF $=$ observed frequency; $E F=$ expected frequency)

|  | Imperfect | Perfect | $\sum$ |
| :--- | :---: | :---: | :---: |
| Accurate | 564 | 586 | 1150 |
| Inaccurate | 96 | 74 | 170 |
| $\Sigma$ | 660 | 660 | 1320 |

Table 47. Comparison of Logit Results by Data Type for Paradigm II, with Data Set C.

|  | OF | EF | OF-EF | $(\mathrm{OF}-\mathrm{EF})^{2}$ | $(O F-E F)^{2} / E F$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Imperfect <br> Accurate | 564 | 575 | -11 | 121 | 0.2104 |
| Imperfect <br> Inaccurate | 96 | 85 | 11 | 121 | 1.4235 |
| Perfect Accurate | 586 | 575 | 11 | 121 | 0.2104 |
| Perfect <br> Inaccurate | 74 | 85 | -11 | 121 | 1.4235 |
|  |  |  |  | $\Sigma$ | 3.2678 |
| df $=1$, alpha $=0.05, x^{2}$ Critical Value $=3.841$ |  |  |  |  |  |
| $\mathrm{X}^{2}=3.2678<3.841$ |  |  |  |  |  |

Table 48. $X^{2}$ Chart Created from Table 47. (OF $=$ observed frequency; $E F=$ expected frequency)
data type (i.e., perfect or imperfect). The previous $X^{2}$ calculations indicate marginal lack of statistical significance for logit. A similar comparison for Data Sets $B$ and $C$ for neural networks is shown in Tables 49 - 52. For Data Set $B$ the computed $X^{2}=125.0648>3.841$, as seen in Table 50. For Data Set $C$ the computed $X^{2}=220.6932>$ 3.841. Both computed values are much greater than the critical value indicating that the neural network or BNN exerts much more influence on the difference between perfect and imperfect prediction accuracies than does logit or $I R$, which had marginal computed $X^{2}$ values.

One conclusion is that there is statistically significant evidence that neural networks may perform better when the data type is imperfect than when it is perfect.

Finally, we test hypothesis two to examine the accuracy of predictions of combat winners based on different paradigm types. This test compares paradigm type <Paradigm I or Paradigm II) against accurate and inaccurate predictions, for both logit and neural networks.

In Tables 53 - 58, logit is tested for each of the three Data Sets A, B, and C. For Data Set A, the computed $x^{2}=62.0232>3.841$ as seen in Table 54 , which is interpreted as statistically significant at the alpha $=0.05$ level of significance. For Data set B, the computed value of $X^{2}=131.5794>3.841$, from Table 56 , which also indicates statistical significance. For Data Set $C$, the computed $X^{2}=187.4418>3.841$. As seen from the

|  | Imperfect | Perfect | $\sum$ |
| :---: | :---: | :---: | :---: |
| Accurate | 454 | 296 | 750 |
| Inaccurate | 57 | 215 | 272 |
| $\Sigma$ | 511 | 511 | 1022 |

Table 49. Comparison of Neural Network Results by Data Type for Paradigm II, with Data Set B.

|  | OF | EF | OF-EF | $(O F-E F)^{2}$ | $(\mathrm{OF}-\mathrm{EF})^{2} / \mathrm{EF}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Imperfect <br> Accurate | 454 | 375 | 79 | 6241 | 16.6427 |
| Imperfect <br> Inaccurate | 57 | 136 | -79 | 6241 | 45.8897 |
| Perfect <br> Accurate | 296 | 375 | -79 | 6241 | 16.6427 |
| Perfect <br> Inaccurate | 215 | 136 | 79 | 6241 | 45.8897 |
|  |  |  |  | $\Sigma$ | 125.0648 |
| df $=1$, alpha $=0.05, \mathrm{x}^{2}$ Critical Value $=3.841$ |  |  |  |  |  |
| $x^{2}=125.0648>3.841$ |  |  |  |  |  |

Table 50. $X^{2}$ Chart Created from Table 49. ( $O F=$ observed frequency; $E F=$ expected frequency)

|  | Imperfect | Perfect | $\sum$ |
| :---: | :---: | :---: | :---: |
| Accurate | 613 | 380 | 993 |
| Inaccurate | 47 | 280 | 327 |
| $\Sigma$ | 660 | 660 | 1320 |

Table 51. Comparison of Neural Network Results by Data Type for Paradigm II, with Data Set C.

|  | OF | EF | OF-EF | $(\mathrm{OF}-\mathrm{EF})^{2}$ | $(\mathrm{OF}-\mathrm{EF})^{2} / \mathrm{EF}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Imperfect <br> Accurate | 613 | 496.5 | 116.5 | 13572.25 | 27.3359 |
| Imperfect <br> Inaccurate | 47 | 163.5 | -116.5 | 13572.25 | 83.0107 |
| Perfect <br> Accurate | 380 | 496.5 | -116.5 | 13572.25 | 27.3359 |
| Perfect <br> Inaccurate | 280 | 163.5 | 116.5 | 13572.25 | 83.0107 |
|  |  |  |  | $\Sigma$ | 220.6932 |
| $\mathrm{df}=1, \quad$ alpha $=0.05, \mathrm{x}^{2}$ Critical value $=3.841$ |  |  |  |  |  |
| $x^{2}=220.6932>3.841$ |  |  |  |  |  |

Table 52. $X^{2}$ Chart Created from Table 51. ( $O F=$ observed frequency; $E F=$ expected frequency)

|  | Paradigm I | Paradigm II | $\Sigma$ |
| :---: | :---: | :---: | :---: |
| Accurate | 82 | 141 | 223 |
| Inaccurate | 67 | 8 | 75 |
| $\Sigma$ | 149 | 149 | 298 |

Table 53. Comparison of Logit Results by Paradigm, for Perfect Data with Data Set A.

|  | OF | EF | OF-EF | $(\mathrm{OF}-\mathrm{EF})^{2}$ | $(O F-E F)^{2} / E F$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Paradigm I <br> Accurate | 82 | 111.5 | -29.5 | 870.25 | 7.8049 |
| Paradigm I Inaccurate | 67 | 37.5 | 29.5 | 870.25 | 23.2067 |
| Paradigm II <br> Accurate | 141 | 111.5 | 29.5 | 870.25 | 7.8049 |
| Paradigm II <br> Inaccurate | 8 | 37.5 | -29.5 | 870.25 | 23.2067 |
|  |  |  |  | $\Sigma$ | 62.0232 |
| df $=1, \quad$ alpha $=0.05, x^{2}$ Critical Value $=3.841$ |  |  |  |  |  |
| $x^{2}=62.0232>3.841$ |  |  |  |  |  |

Table 54. $X^{2}$ Chart Created from Table 53. ( $O F=$ observed frequency; $E F=$ expected frequency)

|  | Paradigm I | Paradigm II | $\sum$ |
| :---: | :---: | :---: | :---: |
| Accurate | 281 | 447 | 728 |
| Inaccurate | 230 | 64 | 294 |
| $\Sigma$ | 511 | 511 | 1022 |

Table 55. Comparison of Logit Results by Paradigm, for Perfect Data with Data Set B.

|  | OF | EF | OF-EE | $(\mathrm{OF}-\mathrm{EF})^{2}$ | $(\mathrm{OF}-\mathrm{EF})^{2} / \mathrm{EF}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Paradigm I <br> Accurate | 281 | 364 | -83 | 6889 | 18.9258 |
| Paradigm I <br> Inaccurate | 230 | 147 | 83 | 6889 | 46.8639 |
| Paradigm II <br> Accurate | 447 | 364 | 83 | 6889 | 18.9258 |
| Paradigm II <br> Inaccurate | 64 | 147 | -83 | 6889 | 46.8639 |
|  |  |  |  | $\Sigma$ | 131.5794 |
| $\mathrm{df}=1$, alpha $=0.05, \mathrm{x}^{2}$ Critical value $=3.841$ |  |  |  |  |  |
| $x^{2}=131.5794>3.841$ |  |  |  |  |  |

Table 56. $X^{2}$ Chart Created from Table 55. $\quad(O F=$ observed frequency; EF $=$ expected frequency)

|  | Paradigm I | Paradigm II | $\sum$ |
| :--- | :---: | :---: | :---: |
| Accurate | 363 | 586 | 949 |
| Inaccurate | 297 | 74 | 371 |
| $\Sigma$ | 660 | 660 | 1320 |

Table 57. Comparison of Logit Results by Paradigm, for Perfect Data with Data Set C.

|  | OF | EF | OF-EF | (OF-EF) $^{2}$ | $(O F-E F)^{2} / E F$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Paradigm I <br> Accurate | 363 | 474.5 | -111.5 | 12432.25 | 26.2007 |
| Paradigm I <br> Inaccurate | 297 | 185.5 | 111.5 | 12432.25 | 67.0202 |
| Paradigm II <br> Accurate | 586 | 474.5 | 111.5 | 12432.25 | 26.2007 |
| Paradigm II <br> Inaccurate | 74 | 185.5 | -111.5 | 12432.25 | 67.0202 |
|  |  |  |  | $\sum$ | 187.4418 |
| df $=1, ~ a l p h a=0.05$, | $x^{2}$ Critical Value $=3.841$ |  |  |  |  |
| $x^{2}=187.4418>3.841$ |  |  |  |  |  |

Table 58. $X^{2}$ Chart Created from Table 57. $\quad(O F=$ observed frequency; $\mathrm{EF}=$ expected frequency)
calculated $X^{2}$ for all three data sets, there is evidence that logit produces statistically significant differences in prediction accuracies for paradigms I and II at the 0.05 level of significance.

To complete the test for hypothesis two, we next test neural networks for the same three data sets. These calculations are shown in Tables 59-64. For Data Set A, the computed $x^{2}=270.9628>3.841$, from Table 60 , which is much greater than the critical $X^{2}$ value. For Data Set $B$, the computed $x^{2}=130.2666>3.841$, from Table 62, again indicating statistical significance. For Data Set C, the computed $X^{2}=193.9414>3.841$, from Table 64, which, once again, indicates strong support for paradigm type influences on prediction accuracies.

All six of the tests for hypothesis two indicate a difference in prediction accuracies based on paradigm type.

While analysis of experimental results does not present statistical ( $X^{2}$ ) evidence in support of the problem statement, hypothesis one for BNN and hypothesis two are supported. The lack of statistical significance, at the alpha $=0.05$ level, of differences between perfect and imperfect data when applied to an LR model, should not be construed as indicating that there is not a problem. The fact that there were any differences at all between perfect and imperfect data point to a need for further research with logit; and, if the logit model had been optimized, perhaps it would have produced significant differences.

|  | Paradigm I | Paradigm II | $\sum$ |
| :--- | :---: | :---: | :---: |
| Accurate | 1 | 143 | 144 |
| Inaccurate | 148 | 6 | 154 |
| $\Sigma$ | 149 | 149 | 298 |

Table 59. Comparison of Neural Network Results by Paradigm, for Imperfect Data with Data Set A.

|  | OF | EF | OF-EF | $(O F-E F)^{2}$ | $(O F-E F)^{2} / E F$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Paradigm I <br> Accurate | 1 | 72 | -71 | 5041 | 70.0139 |
| Paradigm I <br> Inaccurate | 148 | 77 | 71 | 5041 | 65.4675 |
| Paradigm II <br> Accurate | 143 | 72 | 71 | 5041 | 70.0139 |
| Paradigm II <br> Inaccurate | 6 | 77 | -71 | 5041 | 65.4675 |
|  |  |  |  | $\sum$ | 270.9628 |
| df $=1, \quad$ alpha $=0.05, x^{2}$ Critical Value $=3.841$ |  |  |  |  |  |

Table 60. $X^{2}$ Chart Created from Table 59. ( $O F=$ observed frequency; $E F=$ expected frequency)

|  | Paradigm I | Paradigm II | $\Sigma$ |
| :--- | :---: | :---: | :---: |
| Accurate | 292 | 454 | 746 |
| Inaccurate | 219 | 57 | 276 |
| $\Sigma$ | 511 | 511 | 1022 |

Table 61. Comparison of Neural Network Results by Paradigm, for Imperfect Data with Data Set B.

|  | OF | EF | OF-EF | $(O F-E F)^{2}$ | $(\mathrm{OF}-\mathrm{EF})^{2} / \mathrm{EF}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Paradigm I Accurate | 292 | 373 | -81 | 6561 | 17.5898 |
| Paradigm I Inaccurate | 219 | 138 | 81 | 6561 | 47.5435 |
| Paradigm II Accurate | 454 | 373 | 81 | 6561 | 17.5898 |
| Paradigm II Inaccurate | 57 | 138 | -81 | 6561 | 47.5435 |
|  |  |  |  | $\Sigma$ | 130.2666 |
| $\mathrm{df}=1$, alpha $=0.05, \mathrm{X}^{2}$ Critical Value $=3.841$ |  |  |  |  |  |
| $\mathrm{x}^{2}=130.2666>3.841$ |  |  |  |  |  |

Table 62. $X^{2}$ Chart Created from Table 61. ( $O F=$ observed frequency; $E F=$ expected frequency)

|  | Paradigm I | Paradigm II | $\sum$ |
| :--- | :---: | :---: | :---: |
| Accurate | 399 | 613 | 1012 |
| Inaccurate | 261 | 47 | 308 |
| $\Sigma$ | 660 | 660 | 1320 |

Table 63. Comparison of Neural Network Results by Paradigm, for Imperfect Data with Data Set C.

|  | OF | EF | OF-EF | (OF-EF) ${ }^{2}$ | $(O F-E F)^{2} / E F$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Paradigm I <br> Accurate | 399 | 506 | -107 | 11449 | 22.6265 |
| Paradigm I <br> Inaccurate | 261 | 154 | 107 | 11449 | 74.3442 |
| Paradigm II <br> Accurate | 613 | 506 | 107 | 11449 | 22.6265 |
| Paradigm II <br> Inaccurate | 47 | 154 | -107 | 11449 | 74.3442 |
|  |  |  |  | $\Sigma$ | 193.9414 |
| df $=1$, alpha $=0.05, X^{2}$ Critical Value $=3.841$ |  |  |  |  |  |

Table 64. $X^{2}$ Chart Created from Table 63. (OF $=$ observed frequency; $E F=$ expected frequency)

Along with analyzing the problem statement and hypotheses, the research objective included the following goals:
(1) to identify new causal patterns in uncertain and soft data;
(2) to help fill the statistical gap when analyzing uncertain, missing, or soft data;
(3) to demonstrate neural network's capabilities with a complex and nonlinear problem; and,
(4) to examine a potentially credible use of neural networks.

As for the first goal, there were new causal patterns found from this experiment. One pattern is that logit identified key variables that seem to have influence on the winner of battles. These variables are: Attacker Surprise Over Posture Awareness, Number of Artillery Tubes, Relative Combat Effectiveness, Relative Leadership Advantage, Relative Morale Advantage, Relative Intelligence Advantage, and Relative Technology Advantage. If these are key variables in a causal relationship, then their inclusion in military combat models and simulations should be a point for future research.

The second goal was demonstrated by the comparison of imperfect and perfect data for both logit and neural networks. The experiment did indicate that logit performed slightly better with perfect (i.e., clean, filtered) data than with imperfect (i.e., noisy, unfiltered data), although this difference was not significant at the 0.05 level; but, there was statistical evidence for the reverse with the use of neural networks. Thus, one step further has been taken in filling the statistical gap for the type of imperfect data examined in this study. Additional work is still needed for different databases if a purpose of the research is to develop approaches for handling noisy data by the combined use of statistical regression methods and neural networks.

The third and fourth goals were met by the success of applying neural networks to imperfect data for Paradigm II.

## Data

The data coding step was necessary for use with logit. It was not necessary for use with the neural network models. However, the data modeling required for logit produced a rich data taxonomy that can be used for future research. The taxonomy was needed to develop an a priori understanding of the causal relationships between the independent and dependent variables. Thus, the data modeling led to the decision to create artificial data values, such as means or midpoints, for missing or uncertain data. This created the
framework for the final definition of perfect and imperfect data.

## Logit

The logit model development was an exercise in computer programming. To the novice, the logit process may be difficult to learn and is not user friendly. However, the SAS ${ }^{\text {nc }}$ software package was powerful and rich with capabilities for varied statistical analyses. The set up time, however, was longer than expected. The data preparation took several hours, and many more modifications were required over the course of several days.

## Neural Networks

Overall, the neural network was more user friendly than the logit model. The time to learn the logit approach was approximately 50 hours, compared to one hour for the neural network. However, since the structure of the network was trial and error, and since many pre-experiments were tried before this experiment, the overall time of use of the neural network was longer than that of the logit approach. Also, one run of logit may take less than a minute, while one run of the neural network may take several hours.

## CHAPTER 6 <br> CONCLUSIONS

The experiments were successful in providing evidence that addresses the problem statement and the hypotheses. The objective of this study was also met. Thus, the primary conclusion is that use of both logit and neural network models, when analyzing complex data sets, seems warranted. However, for perfect data logit appears to be the model of choice. For noisy data, it appears that neural networks may provide an added capability to the logit approach. Also, when the results of the neural network are in question, the logit could be used to check the validity of the neural network.

The question about whether or not to use neural network models or other statistical methodologies is far from resolved. The above experiments only demonstrate that the use of neural networks is still emerging. In fact, it is possible that within the next decade the statistical community could adopt neural networks as just one other method of regression analysis for special case problems. It is also hoped that this research will add another data point in the further analysis of similar practical applications.

## CHAPTER 7

FUTURE RESEARCH

This research is a direct challenge to the linking of combat winner predictions to attrition, i.e., Paradigm I. Paradigm II, on the other hand, provides an alternative on which to base such predictions that appears to provide greater accuracy. Thus, military computer models and simulations of combat could be suspect if not using strength and skill factors. This is one key area for future research.

Other future research could break down further the basic elements of the human factors variables from Paradigm II in order to determine their specific contribution to battle outcomes.

Another suggestion for future research is to use selforganizing neural networks to analyze a complex data set and account for those rare events where the side that wins does so against all the military wisdom concerning accepted tactics, doctrine and training.

## CHAPTER 8

SUMMARY


#### Abstract

This research was designed to encourage crossdisciplinary dialogue, with a viewpoint that is not strictly that of a neural network researcher, operations research analyst, statistician or other scientific disciplinarian.

The results of these experiments are expected to contribute to the growing science of neural networks, and especially the growing interface between statistics and neural networks, which only recently began to emerge as an important tool within the statistical community.

The examination of combat data using neural networks and logit will allow the military historian and military operations research analyst, as well as military decision makers, to develop a perspective on the importance of environmental, human, and force structure factors in the analysis of combat situations.


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## APPENDIX A

CONVERTED DATA BASE FOR 400 YEARS OF BATTLE


#### Abstract

Input \#1. Ist Width of Front. The attacker 1st Width of Front is divided by that of the defender. The result is a numeric ratio. The rules followed to translate the raw input data are:


a. If attacker and defender are positive, then divide attacker by defender.
b. If the defender is -1 indicating an unknown value for the defender, then indicate this by a 0 value.
c. If attacker and defender are both -1 , indicating that both values are unknown, then leave as -1.

There are no battles when the attacker front is unknown and the defender known.

Input \#2. Defensive Posture Tvpe. The defensive posture type is coded in the historical database as a 0, 1, 2 or 9. The rules followed to translate this data to an interval scale are:
a. 0 means at most one defensive posture type
b. 1 means a combination of postures involving two distinct or separate defensive postures
c. 2 means an average of two or more posture types
d. 3 means more than one posture, but information about whether or not it is able to be averaged is not available

Input \#3. Defender's Primary Defensive Posture Type. The original data was symbolic, which has been translated into an interval scale. The codes used are:
a. 0 means hasty defense
b. 1 means prepared defense
c. 2 means fortified defense
d. 3 means delaying action
e. 4 means withdrawal
f. -1 means unknown or uncertainty in the information

The unknown value of -1 is consistent with other data items measuring uncertainty.

Input \#4. Terrain, The original data was symbolic and has not been converted. Since different terrain can have different effects on attackers and defenders during
different parts of the battle, and since many battles are over large areas, a numeric coding scheme is not appropriate. The symbolic coding system uses a 0 for uncertainty with terrain. The code is composed of a finite combination of letters and $0^{\prime} s$. The coding scheme is:
a. First character:
(1) G means rugged
(2) $R$ means rolling
(3) $F$ means flat
(4) 0 means unknown
b. Second character:
(1) W means heavily wooded
(2) $M$ means mixed
(3) $B$ means bare
(4) D means desert
(5) 0 means unknown

Input \#5. Weather. Weather data are also symbolic. There are four characters used to describe the battlefield weather. A O is used for unknown information. The codes used are:
a. First character:
(1) $W$ means wet
(2) D means dry
(3) 0 means unknown
b. Second character:
(1) H means heavy precipitation
(2) I means light precipitation
(3) 0 means no precipitation and overcast
(4) $S$ means no precipitation and sunny
(5) 0 means unknown
c. Third character:
(1) H means hot
(2) T means temperate
(3) C means cold
(4) O means unknown
d. Fourth character:
(1) E means tropical
(2) D means desert
(3) T means temperate
(4) 0 means unknown

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Input #6. Attacker's Surprise Over Defender's Awareness.
Relative surprise achieved by the attacker is coded along a
range from +3 to -3 in the original data, with 9
representing unknown information. The +3 is complete
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surprise, which means total awareness on the attacker's part and no awareness for the defender. The only data change considered was the unknown value of 9 , which is changed to -9 to place it at a distance from the +3 to -3 range of values. However, there were no 9 values found in the original data. So the degree of uncertainty was not recorded. The coding scheme is:
a. +3 means attacker had complete surprise
b. +2 means attacker had substantial surprise
c. +1 means attacker had minor surprise
d. 0 means there was no surprise
e. -1 means defender had minor surprise
f. -2 means defender had substantial surprise
g. -3 means defender had complete surprise
h. -9 means unknown information about surprise

Using this input as an initial battle data item is open to argument. Some military operations planners argue that it is, and some that it is not, measurable. For this experiment, the assumption is that it is a measurable variable for military operations planning.

Input \#7. Total Personnel Strength of Attacker and
Defender. This is a numeric value, a ratio of the number of attackers divided by the number of defenders for the entire
conflict. Since all original data values are positive, the coding system does not have an uncertainty code.

Input \#8, Initial Personnel Strenoth of Attacker and Defender. This is a numeric value, a ratio of the number of attackers to number of defenders at the start of the conflict. The ratio coding scheme is:
a. If attacker and defender are known, then divide by defender.
b. If attacker is known and defender unknown, use a value of 0 .
c. If attacker is unknown and defender is known, use a value of 0 .
d. If both attacker and defender values are unknown, use -1 .

Input \#9. Horse Cavalry of the Attacker and Defender. This is a numeric value, a ratio of the number of horse cavalry of the attacker divided by those of the defender. This is the first of the technology sensitive data items. The coding scheme used is:
a. If attacker and defender are both known, use a ratio of the attacker divided by the defender value.
b. If the attacker is known and the defender unknown, use 0 .
c. If the attacker is unknown and the defender known, use 0 .
d. If both attacker and defender are unknown, use -1.
e. If there is no information about either, use -9 .

Input \#10. Total Tanks of the Attacker and Defender. This is a numeric value, a ratio of the number of attacker tanks divided by the defender tanks. Due to the diversity in the coding of this data item, the coding scheme is more complex than for previous data items. The coding scheme is:
a. If the attacker and defender are known, use a ratio of the attacker divided by the defender value.
b. If the attacker is known and the defender is unknown, use 0 .
c. If the attacker is unknown and the defender is known, use 0 .
d. If both are 0 or unknown, use -1 .
e. If the attacker is known and the defender is 0 , use +9 .
f. If the attacker is 0 and the defender is known, use 0.1.
g. If both have no value, use -9.

Input \#11. Lite Tanks of the Attacker and Defender. This is a numeric value, a ratio of the lite tanks of the attacker divided by the defender. The same coding scheme used for Input \#10 is used for Input \#11, due to the similarity of the weapon and the original coding of the data.

## Input \#12. Main Battle Tanks of the Attacker and Defender,

 This is a numeric value, a ratio of the main battle tanks of the attacker divided by the defender. The same coding scheme used for Input \#10 is used for Input \#12, due to the similarity of the weapon and the original coding of the data.Input \#13. Number of Artillery Tubes of the Attacker and Defender. This is a numeric value, a ratio of the number of artillery tubes of the attacker divided by those of the defender. The same coding scheme used for Input \#10 is used for Input \#13, due to the similarity of the weapon and the original coding of the data.

Input \#14. Close Air Support Sorties of the Attacker and Defender. This is a numeric value, a ratio of the number of air sorties of the attacker divided by those of the
defender. The same coding scheme used for Input \#10 is used for Input \#14, due to the similarity of the weapon and the original coding of the data.

Input \#15. Relative Combat Effectiveness of Attacker and Defender. This is a scaled numeric value. The coding scheme is:
a. +4 means attacker is very strongly favored
b. +3 means attacker is strongly favored
c. +2 means attacker is favored
d. +1 means attacker is somewhat favored
e. 0 means neither attacker nor defender is favored
f. - 1 means defender is somewhat favored
g. -2 means defender is favored
h. - -3 means defender is strongly favored
i. -4 means defender is very strongly favored
j. -9 means unknown information

Input \#16. Relative Leadership Advantage of Attacker and Defender. The coding scheme is the same as used for Input \#15.

Input \#17, Relative Training Advantage of Attacker and Defender. The coding scheme is the same as used for Input \#15.

Input \#18. Relative Merale Advantage of Attacker and
Defender. The coding scheme is the same as used for Input \#15.

Input \#19. Relative Loqistics Advantare of Attacker and Defender. The coding scheme is the same as used for Input \#15.

Input \#20. Relative Momentum Advantage of Attacker and Defender, The coding scheme is the same as used for Input \#15.

Input \#21. Relative Intelligence Advantage of Attacker and Defender. The coding scheme is the same as used for Input \#15.

Input \#22. Relative Technoloov Adyantage of Attacker and Defender. The coding scheme is the same as used for Input \#15.

Input \#23. Relative Initiative Advantage of Attacker and Defender. The coding scheme is the same as used for Input \#15.

Input \#24. Attacker's Primary Tactical Scheme, Part I. This is a symbolic value. The coding scheme is:
a. FF means frontal attack
b. EE means single envelopment
c. DE means double envelopment
d. FE means feint, or demonstration, or a holding attack
e. DD means defensive plan
f. DO means defensive and/or offensive plan
g. LF means left flank
h. RF means right flank
i. LR means left rear
j. $R R$ means right rear
k. PP means penetration

1. RC means river crossing
m. $\quad \infty$ means unknown
n. 0 also means unknown

Input \#26, Attacker's Primary Tactical Scheme, Part III. This is a symbolic value. The coding scheme is the same as for Input \#24.

Input \#27. Defender's Primary Tactical Scheme, Part I.
This is a symbolic value. The coding scheme is the same as for Input \#24.

Input \#28. Defender's Primary Tactical Scheme, Part II. This is a symbolic value. The coding scheme is the same as for Input \#24.

Input \#29. Defender's Primary Tactical Scheme, Part III. This is a symbolic value. The coding scheme is the same as for Input \#24.

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## APPENDIX B <br> LOGIT PROCEDURES

The attached listings are from $S A S^{m x}$ and consist of the procedural language used to set up and run the logit routine, and include the logit coding of the data, frequency distributions of the data, and several charts used for preliminary data analysis.

```
filename in "/didabal/b/ollie/data/exctotal.csv";
libname out "/didabal/b/ollie/data";
OPTION LS=150 PS=58 replace ;
DATA BATTLE2;
    SET OUT.BATTLE;
* IF LAB54='FF' THEN DLAB54=1;
* ELSE DLAB54=0;
* adD OTHER STATEMENTS HERE ;
    IF LAB12='DSTT' THEN DLAB12=4;
        ELSE IF LAB12= 'DSHT' THEN DLAB12=3;
            ELSE IF IAB12= 'WL'TT' THEN DLAB12=2;
                    ELSE DLAB12=1;
    IF LAB11='RMO' THEN DLAB11=3;
        ELSE IF LAB11='GMO' THEN DLAB11=2;
            ELSE DLAB11=1;
    IF LAB54=`FF' THEN DLAB54=2;
        ELSE DLAB54=1;
    IF LAB55='O' THEN DLAB55=3;
    ELSE IF LAB55='00' THEN DLAB55=3;
        ELSE IF LAB55='EE' THEN DLAB55=2;
            ELSE DLAB55=1;
    IF LAB56='0' THEN DLAB56=2;
        ELSE IF LAB56=`00' THEN DLAB56=2;
            ELSE DLAB56=1;
    IF LAB57= DD' THEN DLAB57=2;
    ELSE IF LAB57='DD' THEN DLAB57=2;
        ELSE DLAB57=1;
    IF LAB58='FF' THEN DLAB58=2;
    ELSE IF LAB58=`O' THEN DLAB58=3;
        ELSE IF LAB58='00' THEN DLAB58-3;
            ELSE DLAB58=1;
    IF LAB59='O' THEN DLAB59=2;
    ELSE IF LAB59='00' THEN DLAB59=2;
```

ELSE DLAB59=1;
IF LAB61=-9 THEN LAB61=0;
IF LAB5=-1 THEN LAB5=0.9397405;
IF LAB10=-1 THEN LAB10=0.8939394;
IF LAB14=-1 THEN LAB14=2.1176215;
IF LAB16=-1 THEN LAB16=1.7773871;
IF LAB26=-1 THEN LABA26=0.2991107;
IF LAB28=-1 THEN LAB28=2.0964664;
IF LAB3O $=-1$ THEN LAB30 $=1.7678282$;
IF LAB32=-1 THEN LAB32=1.4788367;
IF LAB34=-1 THEN LAB34=2.1457696;
IF LAB36=-1 THEN LAB36=1.6123929;
/*ADD STATEMENTS TO CHANGE THE INPUT TO BE PERFECT, I.E., NOT -9;

ARRAY RECODE (*) lab5 lab9 lab10 LAB13 LAB14 LAB16 LAB26 LAB28 LAB30 LAB32 LAB34 LAB36 LAB44 LAB45 LAB46 LAB47 LAB48 LAB49 LAB50 LAB51;

```
Do I =1 to DIM (RECODE)
    IF RECODE (I)=-9 THEN RECODE (I)=.;
END; */
IF LAB5=-9 THEN LAB5=0.9397405;
IF LAB9=-9 THEN LAB9=0.4469697;
IF LAB1O=-9 THEN LAB10=0.8939394;
IF LAB13=-9 THEN LAB13=0.4106061;
IF LAB14=-9 THEN LAB14=2.1176215;
IF LAB16=-9 THEN LAB16=1.7773871;
IF LAB26=-9 THEN LAB26=0.2991107;
IF LAB28=-9 THEN LAB28=2.0964664;
IF LAB3O=-9 THEN LAB3O=1.7678282;
IF LAB32=-9 THEN LAB32=1.4788367;
IF LAB34=-9 THEN LAB34=2.1457696;
IF LAB36=-9 THEN LAB36=1.6123929;
IF LAB44=-9 THEN LAB44=0.1121212;
IF LAB45=-9 THEN LAB45=0.14090901;
IF LAB46=-9 THEN LAB46=0.0242424;
IF LAB47=-9 THEN LAB47=0.2303030;
IF LAB48=-9 THEN LAB48=0.0606061;
IF LAB49=-9 THEN LAB49=0.2166667;
IF LAB50=-9 THEN LAB50=0.0803030;
IF LAB51=-9 THEN LAB51=0.0393939;
```

RUN;
/* PROC MEANS;
VAR lab5 lab9 lab10 dlabll DLAB12 LAB13 LAB14 LAB16 LAB26 LAB28 LAB30 LAB32 LAB34 LAB36 LAB44 LAB45 LAB46 LAB47 LAB48 LAB49 LAB50 LAB51 DLAB54 DLAB55 DLAB56 DLAB57 DLAB58 DLAB59;

RUN; */
PROC PLOT;
PLOT SEQNUM*LAB5;
PLOT SEQNUM*IAB14;
PLOT SEQNUM*IAB44;
PLOT SEQNUM*LAB45;
PLOT SEQNUM*IAB46;
PLOT SEQNUM*LAB47;
PLOT SEQNUM*LAB48;
PLOT SEQNUM*LAB49;
PLOT SEQNUM*LAB50;
PLOT SEQNUM*LAB51;
PLOT SEQNUM*LAB52;
PLOT SEQNUM*LAB16;
PLOT SEQNUM*LAB34;
PLOT SEQNUM*LAB61;
ENDSAS;
/*
PROC FREQ;
tABLES lab5 lab9 lab10 dlab11 DLAB12 LAB13 LAB14 LAB16 LAB26 LAB28 LAB30 LAB32 LAB34 LAB36 LAB44 LAB45 LAB46 LAB47 LAB48 LAB49 LAB50 LAB51 DLAB54 DLAB55 DLAB56 DLAB57 DLAB58 DLAB59;
8/
*PROC FREQ;

* TABLES DLAB11 DLAB12 DLAB54-DLAB59;
* RUN;
/*PROC FREQ data=battle2;
TABLES lab61 lab5 lab9 lab10 lab 11 DLAB12 LAB13 LAB14 LAB16 LAB26 LAB28 LAB30 LAB32 LAB34 LAB36 LAB44 LAB45 LAB46 LAB47 LAB48 LAB49 LAB50 LAB51 DLAB54 DLAB55 DLAB56 DLAB57 DLAB58 DLAB59;

```
    WHERE SEQNUM<=149;
    TITLE" PRE1812 EATTLES";
    RUN; */
/*PROC MEANS DATA=BATTLE2;
    TITLE " MEANS ON VARIABLE";
    RUN; */
*PROC FREQ DATA=BATTLE2;
* TABLES LAB11 LAB12 LAB54-lab59;
* TITLE "FREQENCIES";
* RUN;
proc logistic data=battle2;
    model lab 61=lab5 lab9 lab10 dlab 11 DLAB12 LAB13 LAB14
        LAB16 LAB26 IAB34 LAB44 LAB45 LAB46 LAB47 LAB48 LAB49
        LAB50 LAB51 DLAB54 DI_AB55 DLAB56 DLAB57 DIAB58
        DLAB59/CONVERG=0.025 MAXITER=500
        CTABLE;
        WHERE SEQNUM<=149;
        OUTPUT UT=PRE1812 PRED=PRED;
        TITLE " PERFECT PRE 1812 BATTLES";
        RUN;
proc logistic data=battle2;
    model lab61=lab5 lab 9 lab10 dlab 11 DLAB12 LAB13 LAB14
        LAB16 LAB26 LAB28 LAB30 LAB32 LAB34 LAB36 LAB44 IAB45
        LAB46 LAB47 LAB48 LAB49 LAB50 LAB51 DLAB54 DLAB55
        DLAB56 I_AB57 DLAB55 DLAB59/CTABLE;
        WHERE SEQNUMD149;
        OUTPUT OUT=PST1812 PRED=PRED;
        TITLE "PERFECT POST 1812 BATTLES";
        RINN;
        proc logistic data=battle2;
            model lab61=lab5 lab9 lab10 dlab11 DLAB12 LAB13 LAB14
            IAB16 IAB26 LAB2B LAB30 LAB32 LAB34 IAB36 LAB44 IAB45
            LAB46 LAB47 LAB48 LAB49 LAB50 LAB51 DILAB54 DLAB55
            DLAB56 DIAB57 DLAB58 DLAB59/CTABLEE;
            OUTPUT OUT=TOTAL PRED=PRED;
            TITLE 'PERFECT TOTAL DATASET';
            RUN;
```

ENDSAS;

```
proc logistic data=battle2;
    model lab61=lab5 lab9 lab10 dlab11 DLAB12 LAB13 LAB14
    LAB16 LAB26 LAB28 LAB30 LAB32 LAB34 LAB36 LAB44 LABA45
    LAB46 LAB47 LAB48 LAB49 LAB50 LAB51 DLAB54 DLAB55
    DLAB56 DLAB57 DLAB58 DLAB59 ;
    WHERE SEQNUMD149;
    RUN;
```

ENDSAS;


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